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Distribution Dilemmas

1.1 A Shepherd and his Sheep

Here is a classic puzzle.

An elderly shepherd died and left his entire estate to his three sons. To his first son, whom he favored the most, he bequeathed $\frac{1}{2}$ his flock of sheep, to the second son $\frac{1}{3}$, and to the third son, whom he liked the least, $\frac{1}{9}$ of his flock. (Is there a problem with these proportions?)

Not wishing to contest their father's will, the three sons went to the pasture to begin divvying up the flock. They were alarmed to count a total of 17 sheep! Is there a means for the three sons to successfully carry out their father's wishes?

Taking it Further. Meanwhile, three daughters of a recently deceased shepherdess faced a similar dilemma. Their mother, very wealthy, but also possessing a flawed understanding of fractions, had bequeathed her estate of 495 sheep according to the proportions $\frac{1}{5}$ to her first daughter, $\frac{1}{33}$ to her second, and $\frac{1}{2145}$ to her third! Can her will be successfully honored?

1.2 Iterated Sharing

A group of friends sits in a circle, each with a pile of wrapped candies. (Wrapped candy is used because each piece will be handled by many people before being eaten.) Some people have 20 or more pieces, others none, and the rest some number in between. The distribution is quite arbitrary except for the fact that everyone has been given an *even* number of pieces. A reserve supply is set aside.

The friends now follow these instructions: Give half your candy to the person on your left (and hence receive a supply of candy from the person your right). Do this simultaneously. Now recount your candy supply. If you now have an odd number of pieces, take an extra piece of candy from the reserve supply. This boosts your pile back up to an even number of pieces and everyone is ready to perform the maneuver again.

What happens to the distribution of candy among these friends if this maneuver is performed over and over again? Will people be forever taking extra pieces from the center, so everyone's amount of candy will grow without bound? Or will the distribution stabilize or equalize in some

124 wrapped candies are distributed with a reserve supply of 100 placed in the center.

sense? Will one person end up with all the candy? Might "clumps" of candy move around the circle with each iteration or some strange oscillatory pattern merge? Is it possible to predict what the result will be?

Taking it Further. What happens if instead of adding pieces, you *eat* any odd piece of candy to bring your pile back *down* to an even number? What happens if the sharing pattern is varied; say, you all give half your candy to the person on your left and the other half to the person on your right?