Let $X_1, X_2, \ldots$ be a sequence of identically distributed, pairwise independent nonnegative integer random variables with distribution $P$. Let the expected value be $\mu < \infty$. Let $S_n = \sum_{i=1}^{n} X_i$. It is well-known that $S_n/n$ converges to $\mu$ almost surely. We show that this convergence is effective in $(P, \mu)$. In particular, if $P, \mu$ are computable then the convergence is effective. On the other hand, if the convergence is effective in $P$ then $\mu$ is computable from $P$.

This theorem can be used to show an effective renewal theorem, which then can be used to prove an effective ergodic theorem for countable Markov chains. The last result is a special case of effective ergodic theorems proven by Avigad-Gerhardy-Towsner and Galatolo-Hoyrup-Rojas, but we hope that the direct constructivization of the probability-theory proofs is still useful. (Received September 22, 2010)