When Calderón studied his commutators, in connection with the Cauchy integral on Lipschitz curves, he ran into the bilinear Hilbert transform by dropping an average in his first commutator. He posed the question whether this new operator satisfied any $L^p$ estimates. Lacey and Thiele showed a wide range of $L^p$ estimates in two papers from 1997 and 1999. By dropping two averages in the second Calderón commutator one bumps into the trilinear Hilbert transform. Finding $L^p$ estimates for this operator is still an open question.

In my talk I will discuss $L^p$ estimates for a singular integral operator motivated by Calderón’s second commutator by dropping one average instead of two. I will motivate this operator from a historical perspective and give some comments on how it might be useful in partial differential equations. (Received September 22, 2010)