We prove a structure theorem for the isometry group of an Hermitian map $b : V \times V \rightarrow W$, where $V$ and $W$ are vector spaces over a finite field of odd order. We also present a Las Vegas polynomial-time algorithm to find generators for this isometry group, and to determine its structure. The algorithm can be adapted to construct the intersection of the members in a set of classical subgroups of $GL(V)$, yielding the first polynomial-time solution of this old problem. Our approach develops new computational tools for algebras with involution, which in turn have applications to other algorithmic problems of interest. An implementation of our algorithm in the MAGMA system demonstrates its practicability. (Received July 25, 2010)