In this talk, I will describe the birational geometry of the Hilbert scheme of points in the projective plane. After summarizing results about the ample cone, the effective cone and the stable base locus decomposition, I will describe models that occur while running the Minimal Model Program. These models can be interpreted in terms of moduli spaces of Bridgeland stable objects and can also be constructed via GIT using quiver representations. (Received September 15, 2010)