The purpose of this paper is to prove that the function associated, via the so-called ”sheaf-function dictionary”, to the sheaf of nearby cycles on a certain local model (in the sense of Rapoport-Zink et al.) is essentially a Bernstein function (meaning an element of the Bernstein basis for the Iwahori-Hecke algebra). The local model is one for which the associated algebraic group is a unitary similitude group GU(d). The analogous theorem for the cases of GL(d) and GSp(2d) was done in the 2002 paper ”Nearby cycles for local models of some Shimura varieties” by Haines and Ngô. The method used in the current paper is an adaptation of the method used in the 2002 paper, which was itself inspired by a conjecture of Kottwitz. (Received September 21, 2010)