On page 335 in his lost notebook, Ramanujan recorded without proofs two identities involving finite trigonometric sums and doubly infinite series of Bessel functions, which are intimately connected with the classical circle and divisor problems. In this talk, we discuss the proofs of them under two interpretations for the double series. Moreover, by extending the same ideas, we present three new identities, which involve analogous Bessel series. For example, one of our identities is for \( \sum_{nm \leq x} \cos(2\pi n\theta) \sin(2\pi m\sigma) \). (Received September 14, 2010)