Using the \( p \)-Group Generation Algorithm to Determine Extensions of \( D_4 \) by \( C_2 \times C_2 \times C_2^n \). Preliminary report.

Let \( p \) be a prime and let \( D_4 \) represent the dihedral group of order 8. Michael Bush used the \( p \)-group generation algorithm to determine possible presentations for the Galois groups of the maximal unramified 2-extensions of several imaginary quadratic fields. His work with \( \mathbb{Q}(\sqrt{-445}) \), \( \mathbb{Q}(\sqrt{-1015}) \), and \( \mathbb{Q}(\sqrt{-1595}) \) led us to discover that their Galois groups are members of a larger family of group extensions of \( D_4 \) by \( C_2 \times C_2 \times C_2^n \) with respect to a certain action. Using Magma, we conjectured that for each \( n \geq 3 \), there are 8 distinct such extensions of \( D_4 \) by \( C_2 \times C_2 \times C_2^n \). Furthermore, these 8 groups appear to form 4 pairs such that the groups in each pair have isomorphic subgroup lattices and under this isomorphism, corresponding proper subgroups and quotients are isomorphic. Thus, distinguishing these groups is quite difficult. In this talk, we will discuss how we are currently utilizing the \( p \)-group generation algorithm by hand to compute presentations for these extensions and thereby prove our conjecture. (Received July 26, 2010)