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**Jason D Yust\*** ([jason.yust@gmail.com](mailto:jason.yust@gmail.com)), 11 Country Club Circle, Tuscaloosa, AL 35401. *The Mathematics of Contrapuntal Hierarchy in Music.*

The work of Heinrich Schenker (1868-1935) describes the hierarchic organization of tonal harmony in eighteenth and nineteenth century music. A class of networks called maximal outerplanar graphs (MOPs) or triangulations of the  $n$ -gon, combinatorially equivalent to binary plane trees, accurately describes many forms of musical hierarchy, including simplest structures of Schenkerian theory. But they do not suffice for complete analyses because they do not account for the most basic aspect of counterpoint, the possibility of simultaneous semi-independent activity in multiple voices. Schenker's own concept of counterpoint is unique because his theory requires that musical intervals be considered melodic and harmonic at the same time (according to the principles of unfolding and linear progression). Schenker's theory therefore implies a new form of "contrapuntal hierarchy" which can be represented by an expanded class of networks: 2-trees. 2-trees are closely related to MOPs, so we can draw on the geometry of simple hierarchy, the Stasheff polytope or associahedron, to describe the relationships between different contrapuntal hierarchies. Examples from J.S. Bach's Partitias illustrate contrapuntal hierarchies in music analysis. (Received September 18, 2010)