Many social contagions require multiple contacts or sources of activation to spread. Examples include adoption of new technologies, participation in social movements, etc. Such processes are called complex contagions. These have so far been modeled on graphs by threshold systems with fixed thresholds.

However, a fixed threshold does not capture the dynamics of systems involving varying disease immunity, evolving psychological factors in social contagions, etc. We generalize the notion of threshold systems to incorporate the effects of dynamics and history. Namely, we have increasing, decreasing, and mixed thresholds that better reflect realistic dynamics; we call this generalization an evolving threshold dynamical system (ET-DS).

Using the framework of graph dynamical systems, we characterize the long-term behavior of dynamic threshold systems, enumerate the number of limit sets, describe the phase spaces of such systems and, in the process, elucidate topological conjugacies between dynamical system maps of these threshold functions. We then consider ET-DSs on some specific graph classes, deriving explicit formulae for the number of fixed points. (Received August 03, 2010)