Coin set extensions in the greedy change-making problem.

Given a finite sequence $A = (1, a_1, \ldots, a_k)$ of positive integer coin denominations, we can make change for a positive integer amount $x$ using the greedy algorithm, that is, by iteratively choosing the largest coin value $a_i \leq x$, then the largest coin $a_i \leq x - a_i$, and so on. Call a coin set orderly if, for every positive integer $x$, the greedy algorithm makes change for $x$ with the fewest possible number of coins. Call a coin set $B$ an extension of a coin set $A$ if $B \supset A$ and all coins in $B - A$ are larger than the largest coin in $A$. Call a coin set an obstruction if it cannot be extended to an orderly coin set. We present a new characterization of orderly coin sets, and use this characterization to find simple conditions for when a one-coin extension is orderly. We also present a series of sufficient conditions to determine if a coin set is an obstruction, and we fully characterize all obstructions of length four. (Received September 21, 2010)