Thin spanning trees, conductances, nowhere zero flows, and the traveling salesman problem.

A spanning tree $T$ in a graph $G$ is $\epsilon$-thin if $T$ contains at most an $\epsilon$ fraction of the edges of every cut. Goddyn’s conjecture says that every $f(\epsilon)$-edge-connected graph contains an $\epsilon$-thin tree for a suitable function $f$. In this talk, we discuss this conjecture and variants of it, and its implications for nowhere zero 3-flows and for the approximability of the asymmetric traveling salesman problem. In particular, we show that, if the graph is $(c \log(n) / \log \log(n))$-edge-connected, one can select conductances in a corresponding electrical network so that a random spanning tree is $\epsilon$-thin with high probability. We also show that, if we replace the spanning tree requirement by simply having a linear number of edges then Goddyn’s conjecture can be proved. (Received September 17, 2010)