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Michael Christ* (mchrist@math.berkeley.edu), Department of Mathematics, University of California, Berkeley, CA 94720. *Upper Bounds for Multilinear Sublevel Sets.*

Let $\ell_j : \mathbb{R}^d \rightarrow \mathbb{R}^{d_j}$ be surjective linear transformations, let $P : \mathbb{R}^d \rightarrow \mathbb{R}$ be a real-valued polynomial, let B be a ball in \mathbb{R}^d . The associated sublevel sets are

$$E_\varepsilon(P, g_1, \dots, g_n) = \left\{ y \in B : \left| P(y) - \sum_{j=1}^n g_j(\ell_j(y)) \right| < \varepsilon \right\},$$

where $g_j : \mathbb{R}^{d_j} \rightarrow \mathbb{R}$ are arbitrary measurable functions. We study upper measure bounds of the form

$$|E_\varepsilon(P, g_1, \dots, g_n)| \leq \rho(\varepsilon)$$

which are uniform over all measurable functions g_j , with $\rho(\varepsilon) \rightarrow 0$ as $\varepsilon \rightarrow 0$. Such bounds would be implied by conjectured multilinear oscillatory integral inequalities. We prove the sublevel set bounds under the natural nondegeneracy hypothesis on P , supplemented by an auxiliary rationality hypothesis. The analysis involves an alternative notion called finitely witnessed nondegeneracy, and relies on a variant of Szemerédi's theorem due to Furstenberg and Katznelson. (Received September 22, 2009)