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**Julia F. Knight\***, knight.1@nd.edu, and **Karen Lange**, klange1@nd.edu. *Generalized power series and real closed fields: Part II.*

Mourgues and Ressayre showed that every real closed field has an integer part. Their construction involves mapping the given real closed field  $R$  isomorphically onto a truncation closed sub-field of the field  $k\langle\langle G\rangle\rangle$ , where  $G$  is the natural value group of  $R$  and  $k$  is the residue field. We refer to the image of  $r \in R$  as its *development*. If  $G$  has cardinality  $\kappa$ , then the developments may have arbitrary ordinal length less than  $\kappa^+$ . We consider the case where  $R$  is countable, and we list the elements of a transcendence base for  $R$  over  $k$ — $r_1, r_2, \dots$ . In terms of this list, the Mourgues and Ressayre construction becomes canonical. Let  $R_n$  be the real closure of  $R_n(r_1, \dots, r_n)$ . By a result of Shepherdson, the elements of  $R_1$  have developments of length at most  $\omega$ . We show that elements of  $R_n$  have developments of length at most  $\omega^{\omega^{(n-1)}}$ . Thus, the elements of  $R$  have developments of length less than  $\omega^{\omega^\omega}$ . These bounds are sharp. Letting  $G$  be generated by a single infinitesimal, we produce a sequence of elements  $r_1, r_2, \dots$  such that for each  $n \geq 1$ ,  $R_n$  contains an element whose development has length  $\omega^{\omega^{(n-1)}}$ . (Received September 15, 2009)