1046-65-1206 **Li-Lian Wang*** (lilian@ntu.edu.sg), Division of Mathematical Sciences, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore, Singapore. *Generalized Prolate Spheroidal Wave Functions and Spectral Methods on Quasi-uniform Grids.*

The Slepian series (i.e., the prolate spheroidal wave functions of order zero) has been proven to be an optimal tool for approximating bandlimited functions. Recently, there has been a growing interest in developing methods, including spectral/spectral-element methods, wavelets and infinite elements, using the Slepian series basis functions, which exhibit some remarkable advantages over their polynomial-based counterparts.

In this talk, we will define a family of generalized prolate spheroidal wave functions (GPSWFs), which are orthogonal with respect to the Gegenbauer weight function $(1-x^2)^\alpha$. The GPSWFs generalize the Slepian series to order $\alpha > -1$, and can also be viewed as a generalization of the Gegenbauer polynomials, but oscillate more uniformly over $(-1,1)$. Two special cases: $\alpha = 0, -\frac{1}{2}$ are of particular importance, and the GPSWFs of order $-\frac{1}{2}$ are closely related to Mathieu functions, which form a natural basis functions for PDEs in elliptic geometries.

We will present some properties and a set of GPSWF approximation results. We will also develop spectral methods on quasi-uniform grids based on the GPSWFs, and discuss their applications to time-harmonic scattering problems. (Received September 15, 2008)