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Vladimir G. Pestov* (pestovv@member.ams.org), Department of Mathematics and Statistics, University of Ottawa, 585 King Edward Avenue, Ottawa, Ontario K1N 6N5, Canada. *Matrix models for discrete and topological groups.*

A discrete group Γ admits a matrix model (or is hyperlinear) if Γ embeds into an ultraproduct of unitary groups $U(n)$, $n \in \mathbb{N}$ equipped with the normalized Hilbert-Schmidt distance, with regard to some (any) non-principal ultrafilter on natural numbers, formed in the same way as ultraproducts of Banach spaces. (The expression "matrix model" comes from Mathematical Physics by way of Operator Algebras, but the above property of Γ can indeed be stated in terms of model theory of continuous structures in the sense of Ben Yaacov, Berenstein, Henson, and Usvyatsov.) It remains unknown whether every group admits a matrix model. The importance of this question comes from its close link to a number of open problems in various areas of mathematics, above all Connes' Embedding Conjecture for Operator Algebras, but also solvability of equations in groups, and others. We will survey some known and new results and open problems, and also discuss a natural extension of matrix models to metric groups, which allows for the first time to bring into the picture a number of well-known infinite-dimensional groups, such as the group of measure preserving transformation of a standard Lebesgue measure space, full groups of measure-preserving equivalence relations, etc. (Received September 14, 2008)