

1023-Z1-524

Douglas G. Burkholder* (burkholderd@lrc.edu), Box 7274, Lenoir-Rhyne College, Hickory, NC 28603. *Planarizing Non-Planar Polygons.*

Recall that the midpoints of a (not necessarily planar) quadrilateral form a parallelogram. Thus, the midpoint rule planarizes quadrilaterals. In 1960, Jesse Douglas found two constructions using the median lines which planarize pentagons. In all three of these cases, the process of planarizing created polygons which, to quote Jesse Douglas, are "as regular as possible." Specifically, these three constructions create affine regular polygons. Here we consider n -gons consisting of any sequence of n points in any vector space. Any set of n -weights can be used to generate a new n -gon by sequentially calculating the weighted average of these n points. For example, the midpoint rule is obtained by applying the weights $\frac{1}{2}, \frac{1}{2}, 0, 0$. Here we determine all possible weights which will planarize arbitrary polygons. In the process, we determine that for every n , there exist an infinite number of different planarizing constructions. If a set of weights planarizes all polygons in Euclidean 3-space, then the weights planarizes all polygons in every vector space. Moreover, the planarization process always regularizes. That is, every planarization produces affine regular polygons. (Received September 15, 2006)