One approach taken to understanding turbulence is the development of asymptotic equations for the movement of thin tube-like filaments with intense vorticity. I will review past results on local self-induced motion and motion of a filament whose centerline deviates initially from a given smooth curve by only a small-amplitude but short-wavelength displacement.

The primary focus of the talk will be to discuss the dynamics of a vortex line immersed in a three-dimensional averaged fluid velocity field governed by the Lagrangian averaged Navier-Stokes (LANS-α) equations. The LANS-α equations are a set of equations designed to capture the large scale dynamics of the Navier-Stokes equations by averaging, or filtering, motion at spatial scales smaller than a chosen parameter $\alpha > 0$. Preliminary results show that Lagrangian averaging replaces the core structure of the filament. Consequently, the Biot-Savart integral is no longer singular and the induced velocity at any point in the presence of a vortex line can be determined for $\alpha > 0$. The main tools come from PDEs and asymptotic analysis. (Received September 22, 2006)