Borcherds described the exponents $a(n)$ in product expansions $f = q^h \prod_{n=1}^{\infty} (1 - q^n)^{a(n)}$ of meromorphic modular forms with a Heegner divisor. His description gives the order of vanishing at infinity $h$ of $f$ as a generalized class number. We give $p$-adic formulas for $h$ in terms of generalized traces over the zeros and poles of $f$. Specializing to the case of the Hilbert class polynomial $f = \mathcal{H}_d(j(z))$ yields $p$-adic formulas for class numbers that generalize past results of Bruinier, Kohnen, and Ono. (Received July 28, 2006)