We develop the theory of polyhedral theta functions. Some (but not all) of these theta functions, defined over polyhedral cones, are in fact modular forms. Upon taking limits of these polyhedral theta functions, we obtain certain relations for solid angles of polytopes. Other relations, that are still conjectural, were obtained for sums of solid angles of certain polytopes and these relations are experimentally discovered. (Received September 04, 2006)