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David Clampitt* (david.clampitt@yale.edu), Yale University, Department of Music, P. O. Box 208310, New Haven, CT 06520-8310. *Mathematical Aspects of Pairwise Well-formed Scales.*

Pairwise well-formed scales include important collections in world music. Pairwise well-formedness is a second-order notion, based upon that of well-formedness. A scale is *well-formed* if it is generated by an interval of constant span and size, i.e., if all the notes of the scale may be linked together in a chain, where the links are intervals of the same size that span the same number of scale steps. That is, for θ real and an integer $N > 1$, consider $S = \{n\theta - \lfloor n\theta \rfloor \mid 0 \leq n < N\}$. Then $S = \{s_0, s_1, \dots, s_{N-1}\}$, where $0 = s_0 < \dots < s_{N-1}$, is well-formed if and only if there exists a unit $u \pmod N$ such that $\mu : \mathbb{Z}_N \rightarrow \mathbb{Z}_N$ maps z to $uz \pmod N$ where $s_{\mu(z)} = z\theta - \lfloor z\theta \rfloor$. A scale is *non-degenerate well-formed* if it is well-formed and its step intervals (differences $s_{i+1} - s_i$) come in two sizes. A scale is *pairwise well-formed* (PWWF) if, when any pair of its step-interval sizes is taken to be equivalent, the resulting pattern is that of a non-degenerate well-formed scale. (Received September 15, 2006)