Combinatorial symmetry of the 24-cell via matroids.

The Coxeter group $F_4$ is the symmetry group of the 24-cell, a regular 4-dimensional solid. Let $S$ be the set of all vectors normal to all the hyperplanes of symmetry of the 24-cell, and let $M(S)$ be the associated matroid over the rational numbers. (The set $S$ is the root system $F_4$.) We compare the geometric symmetry of the 24-cell with the combinatorial symmetry of the associated matroid by computing the automorphism group of $M(S)$. This automorphism group includes all of the geometric symmetries in addition to a non-geometric action, which nevertheless has a geometric interpretation. We give the structure of this group and compare it to $F_4$.

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