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Adam J Hammett* (hammett@math.ohio-state.edu), 231 W. 18th Avenue, Columbus, OH ,
and **Boris G Pittel**, 231 W. 18th Avenue, Columbus, OH. *On the Likelihood of Comparability in Bruhat Order.*

Two permutations of $[n] := \{1, 2, \dots, n\}$ are comparable in the *Bruhat order* if one can be obtained from the other by a sequence of transpositions decreasing the number of inversions. We show that the total number of pairs of permutations (π, σ) with $\pi \leq \sigma$ is of order $(n!)^2/n^2$ at most. Equivalently, if π, σ are chosen uniformly at random and independently of each other, then $Pr\{\pi \leq \sigma\}$ is of order n^{-2} at most. By a direct probabilistic argument we prove $Pr\{\pi \leq \sigma\}$ is of order $(0.708)^n$ at least, so that there is currently a wide qualitative gap between the upper and lower bounds.

For the *weak Bruhat order* “ \preceq ” – when only adjacent transpositions are admissible – we use a non-inversion set criterion to prove that $P_n^* := Pr\{\pi \preceq \sigma\}$ is submultiplicative, thus showing existence of $\rho = \lim \sqrt[n]{P_n^*}$. We demonstrate that ρ is 0.362 at most. Moreover, we prove the lower bound $\prod_{i=1}^n (H(i)/i)$ for P_n^* , where $H(i) := \sum_{j=1}^i 1/j$. In light of numerical experiments, we conjecture that for each order the upper bound is qualitatively close to the actual behavior. (Received September 26, 2006)