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44115. θ -Frames in R^n . Preliminary report.

Let $0 < \theta < \pi$ and $(R^n, \|\cdot\|)$, $n \geq 2$, be the Euclidean space with the standard inner product coordinatized by the standard basis. **Definition.** A θ -frame in R^n , $n \geq 2$ is an ordered basis of R^n , $\{\vec{x}_i\}_{i=1}^n$, $n \geq 2$, such that $\|\vec{x}_i\| = 1$, $1 \leq i \leq n$, and $(\vec{x}_i, \vec{x}_j) = \cos\theta$, $i \neq j$. It is clear every θ -frame F_θ is canonically associated with a unique $n \times n$ matrix F'_θ .

Theorem 1 A θ -frame in R^n , $n \geq 2$ exists if and only if $\cos\theta > -\frac{1}{(n-1)}$

Theorem 2 If $m > n$, each θ -frame in R^n admits an extension to a θ -frame in R^m , R^m if $\cos\theta > -\frac{1}{(m-1)}$.

Theorem 3 For each θ , $\cos\theta > -\frac{1}{(n-1)}$, there is a unique frame s_θ such that the matrix s_θ^1 is positive definite. The matrix of a θ -frame is normal if and only if the transpose of the matrix is associated with a θ -frame.

Several interesting geometric properties of θ -frames are obtained.

The motivation of the investigation arose in answering a problem on θ -frames in R^3 raised by an engineer. (Received July 19, 2005)