In 1933 a family of skew Paley-Hadamard difference sets was described by Paley using matrix languages. During the last 70 years, no other skew Paley-Hadamard difference sets were found. It was conjectured that there are no further examples of skew Paley-Hadamard difference sets. This conjecture was proved to be true for the cyclic case by Kelly in 1954 and Johnson in 1966. Further efforts in this direction were made by Camion and Mann in 1972, Jungnickel in 1990, and Chen, Xiang, and Sehgal in 1994. However, the problem as to whether there are other skew Paley-Hadamard difference sets remains open.

We have just discovered a new family of skew Paley-Hadamard difference sets. This disproves the longstanding conjecture, and may lead to a new direction in searching for further skew Paley-Hadamard difference sets.

In this talk, we shall introduce the new family of skew Paley-Hadamard difference sets, and talk about their $p$-ary linear codes.

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