Sonar inversion strategies.

'Sonar transforms' are certain spherical mean operators arising in reflective tomography. The canonical example of such an operator is the mapping which assigns to functions in $\mathbb{R}^d$ their integrals over the spheres centered on $\mathbb{R}^{d-1}$. We will refer to this mapping as the sonar transform on $(\mathbb{R}^d, \mathbb{R}^{d-1})$ or 'Euclidean sonar' for short. More generally, let $M$ be a Riemannian manifold and let $C \subset M$ be a connected sub-manifold of co-dimension one. We can define the sonar transform on $(M, C)$ to be the mapping which integrates functions on $M$ over the $M$-spheres centered on $C$.

The focus of the talk will lie on the current state of the sonar theory and the possibilities for its future development. In the beginning I will use simple sonar spaces to illustrate the sonar inversion techniques that are currently scattered in the literature. After a historical introduction I will outline a new approach to sonar inversion based on the connection with weighted Radon transforms. The talk will be concluded with a brief comparative analysis of the sonar inversion strategies and a few remarks about possible extensions of the theory. (Received September 29, 2004)