Let $A$ be a bounded set of $\mathbb{R}^{n+r}$ definable in an o-minimal expansion $\mathcal{S}$ of the reals. Assume that for all $a \in \mathbb{R}^r$, the fibers $A_a$ are either empty or compact. The Hausdorff limit of any sequence of fibers is known to be $\mathcal{S}$-definable. We give a formula that bounds the Betti numbers of such a limit in terms of Betti numbers of simple sets defined from the fibers of $A$. In particular, this leads to effective upper bounds in the semialgebraic setting. In the Pfaffian setting, Gabrielov showed that the structure generated by Pfaffian functions could be described using relative closures, and the present work can be applied to give explicit upper-bounds on the Betti numbers of such relative closures. (Received October 04, 2004)