A characterization of which sequences can be the Hilbert function of a finite set of distinct points in $\mathbb{P}^n$ (called valid Hilbert functions for $\mathbb{P}^n$) follows from the work of Macaulay, Hartshorne, and others. Although Hilbert functions of complete intersections are well-known, Hilbert functions of subsets of complete intersections have not been classified, even for the simplest cases. Let $d_1 \leq d_2 \leq \cdots \leq d_n$ be positive integers and $H$ be a valid Hilbert function for $\mathbb{P}^n$. We wish to determine if there exists some reduced zero-dimensional complete intersection $C.I.(d_1, \ldots, d_n)$ which contains a subset whose Hilbert function is $H$.

The special case of this problem where the ideal of the complete intersection is generated by products of linear forms follows from the work of Clements and Lindström. In general, this problem was completely answered for the case $n = 2$ in my M.Sc. thesis and I currently have partial results for $n = 3$. We will discuss these results as well as consider the complications that have arisen in working on this research problem. We will conclude with applications to subsets with the Cayley-Bacharach Property and extremal subsets similar to $\text{Sub}_d(H)$. This work will be included in my Ph.D. dissertation. (Received September 28, 2004)