

1147-52-765

Allan Berele and **Stefan Catoiu*** (scatoiu@depaul.edu), Department of Mathematics, DePaul University, Chicago, IL. *Fermat-Torricelli Theorem in Convex Geometry.*

We study a generalization of the Euclidean triangle which we believe to be the right one for convex geometry. To illustrate its power, our main result provides the first instance when a major theorem in Euclidean geometry – Fermat-Torricelli theorem – finds its greatest generality in convex geometry.

A point that minimizes the sum of distances to the vertices of a triangle (Fermat point) is the same as one through which pass three equiangular affine diameters (Fermat-Ceder point). A generalized deltoid is a triangle whose sides are disjoint, outwardly-looking arcs of convex curves. We extend the Fermat-Ceder point of a triangle to a Fermat-Ceder point of a generalized deltoid. Fermat-Torricelli theorem in convex geometry says that each generalized deltoid admits a unique Fermat-Ceder point.

As an application, we show that the Fermat-Ceder points for the continuous families of affine diameters, area-bisecting lines, and perimeter-bisecting lines are unique for every triangle, and non-unique for every pentagon. In the case of quadrilaterals, the uniqueness of the affine Fermat-Ceder point holds precisely for all non-trapezoids, the one for the area Fermat-Ceder point holds for all quadrilaterals, and the one for the perimeter Fermat-Ceder point is still open. (Received January 29, 2019)