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Mauro Di Nasso* (mauro.di.nasso@unipi.it), Dipartimento di Matematica, Largo B. Pontecorvo 5, 56127 Pisa, Italy. *Ultrafilters as nonstandard points: some new applications in Ramsey Theory.*

In Ramsey Theory one studies the notion of partition regularity (PR): A family G is PR on X if for every finite coloring $X = C_1 \cup \dots \cup C_r$ there exists a *monochromatic* $A \in G$, i.e., $A \subseteq C_i$ for some i . In this setting, *ultrafilters* play an instrumental role: A family G is PR on X iff there exists an ultrafilter U on X such that every element of U includes some $A \in G$. By using the *nonstandard extension* *X , ultrafilters on X can be represented as points $\xi \in {}^*X$. I will present a nonstandard technique grounded on that observation, which has been recently used to prove new results about the PR of Diophantine equations. (An equation $P(x_1, \dots, x_n) = 0$ is PR if the set of its solutions $\{\{a_1, \dots, a_n\} \mid P(a_1, \dots, a_n) = 0\}$ is PR.) As examples, I will show that $X^2 + Y^2 = Z$ and $X + Y = Z^2$ are *not* PR.

In the second part of the talk, I will briefly discuss the (discrete) topological dynamics as given by the hypernatural numbers ${}^*\mathbb{N}$ endowed with the shift operator $S : \xi \mapsto \xi + 1$, and present an alternative nonstandard proof of *van der Waerden's Theorem*: In any finite coloring of the natural numbers there exist monochromatic arithmetic progressions of arbitrary length. (Received January 14, 2019)