Point-Line Arrangements. Preliminary report.
The famous Szemerédi-Trotter theorem states that any arrangement of $n$ points and $n$ lines in the plane determines $O\left(n^{4 / 3}\right)$ incidences, and this bound is tight. In this talk, we present some Turán-type results for point-line incidences. Let $\mathcal{L}_{1}$ and $\mathcal{L}_{2}$ be two sets of $t$ lines in the plane and let $P=\left\{\ell_{1} \cap \ell_{2}: \ell_{1} \in \mathcal{L}_{1}, \ell_{2} \in \mathcal{L}_{2}\right\}$ be the set of intersection points between $\mathcal{L}_{1}$ and $\mathcal{L}_{2}$. We say that $\left(P, \mathcal{L}_{1} \cup \mathcal{L}_{2}\right)$ forms a natural $t \times t$ grid if $|P|=t^{2}$, and conv $(P)$ does not contain the intersection point of some two lines in $\mathcal{L}_{i}$, for $i=1,2$. For fixed $t>1$, we show that any arrangement of $n$ points and $n$ lines in the plane that does not contain a natural $t \times t$ grid determines $O\left(n^{\frac{4}{3}-\varepsilon}\right)$ incidences, where $\varepsilon=\varepsilon(t)$. We also provide a construction of $n$ points and $n$ lines in the plane that does not contain a natural $2 \times 2$ grid and determines at least $\Omega\left(n^{1+\frac{1}{14}}\right)$ incidences. This is joint work with Andrew Suk. (Received August 13, 2019)

