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Colin Defant* (cdefant@princeton.edu), **Michael Engen** and **Jordan A Miller**. *Lassalle's Sequence Counts Uniquely Sorted Permutations.*

Consider the sequence $(A_m)_{m \geq 1}$ satisfying $A_1 = 1$ and

$$A_m = (-1)^{m-1} C_m + \sum_{j=1}^{m-1} (-1)^{j-1} \binom{2m-1}{2m-2j-1} A_{m-j} C_j$$

for $m \geq 2$, where C_n is the n^{th} Catalan number. Lassalle gave an algebraic proof that the terms in this sequence are positive and increasing, settling a conjecture of Zeilberger. We show that A_{k+1} is the number of permutations $\pi \in S_{2k+1}$ satisfying $|s^{-1}(\pi)| = 1$, where s is West's stack-sorting map. This result follows from a more general bijection between certain weighted set partitions and relatively new combinatorial objects called *valid hook configurations*. These objects were introduced in order to provide a method, which we will describe, for computing $|s^{-1}(\pi)|$ for any permutation π . We discuss connections between valid hook configurations and cumulants arising in free probability theory. Finally, let $A_{k+1}(\ell)$ denote the number of permutations $\pi = \pi_1 \pi_2 \cdots \pi_{2k+1} \in S_{2k+1}$ satisfying $|s^{-1}(\pi)| = 1$ and $\pi_1 = \ell$. We show that the sequence $A_{k+1}(1), A_{k+1}(2), \dots, A_{k+1}(2k+1)$ is symmetric, and we conjecture that it is log-concave. (Received September 03, 2018)