1145-05-1735 **Huy Tuan Pham*** (huypham@stanford.edu). *Tower-type bounds for Roth's theorem with popular differences.*

A famous theorem of Roth states that for any $\alpha > 0$ and n sufficiently large in terms of α, any subset of $[n]$ with density α contains a 3-term arithmetic progression. Green developed an arithmetic analogue of Szemerédi's regularity lemma to prove that not only is there one arithmetic progression, but in fact there is some integer $d > 0$ for which the density of 3-term arithmetic progressions with common difference d is at least roughly what is expected in a random set with density α. In particular, for any $\in > 0$, there is some $n_\in$ such that for all $n > n_\in$ and any subset A of $[n]$ with density α, there is some integer $d > 0$ for which the number of 3-term arithmetic progressions in A with common difference d is at least $(\alpha^3 - \epsilon)n$. We prove that $n_\in$ grows as an exponential tower of 2's of height on the order of $\log\left(\frac{1}{\in}\right)$. We show that the same is true if we replace the interval $[n]$ by any abelian group of odd order n. These results are the first applications of regularity lemmas for which the tower-type bounds are shown to be necessary.

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