1145-03-1492 **Philipp C K Hieronymi*** (phierony@illinois.edu), 1409 W Green Street, Urbana, IL 61821.
Decidability, Diophantine Approximation and Ostrowski numeration systems.

It has long been known that the theory of the expansion $(\mathbb{R},<,+,\mathbb{Z})$ of the ordered additive group of real numbers by the set of integers is decidable. Arguably due to Skolem, the result can be deduced easily from Buechi's theorem on the decidability of monadic second order theory of one successor, and was later rediscovered independently by Weispfenning and Miller. However, a consequence of Goedel's incompleteness theorem states that when expanding this structure by a symbol for multiplication, the theory of the resulting structure $(\mathbb{R},<,+,\cdot,\mathbb{Z})$ becomes undecidable. This observation gives rise to the following question: How many traces of multiplication can be added to $(\mathbb{R},<,+,\mathbb{Z})$ without making the theory undecidable? For $b \in \mathbb{R}$, let $f_b : \mathbb{R} \to \mathbb{R}$ be the function that takes x to bx. Then the theory of $(\mathbb{R},<,+,\mathbb{Z},f_b)$ is decidable if and only if b is quadratic. The proof rests on the observation that many of the Diophantine properties (in the sense of Diophantine approximation) of b can be coded in these structures. In particular, the Ostrowski numeration system based on b is definable in this structure. (Received September 22, 2018)