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Trung Hoa Dinh* (trunghoa.math@gmail.com), Department of Mathematics and Statistics, University of North Florida, 1 UNF Drive, Jacksonville, FL 32224. *Some open problems in the theory of matrix means and matrix functions.*

The problem to find a matrix analog of the geometric mean of non-negative numbers was a long standing since the product of two positive semi-definite matrices is not a such one. In 1975, Pusz and Woronowitz solved this problem and showed that the geometric mean $A\sharp B := A^{1/2}(A^{-1/2}BA^{-1/2})^{1/2}A^{1/2}$ of two positive definite matrices A and B is the unique solution of the matrix Riccati equation $XA^{-1}X = B$. Consequently, Ando and Kubo (1980) developed an axiomatic theory of operator means on the set of positive semi-definite matrices and showed that there exists an affine order-isomorphism from the class of operator means onto the class of positive operator monotone functions which were introduced by Loewner (1930). In 1996, Petz proved that there is a correspondence between monotone metrics and operator means in the sense of Kubo and Ando, and hence, connected three important theories in matrix analysis and quantum information theory.

In this talk we introduce some picture of matrix means and a general approach of matrix convexity. The problem of characterization of matrix functions by inequalities is also discussed. The topics and open problems are well-suited for undergraduate students with background in Linear Algebra and Analysis. (Received July 18, 2017)