1135-11-825 **Robert Schneider*** (`robert.schneider@emory.edu`). *Toward an arithmetic of partitions.*
Much like the positive integers $\mathbb{Z}^+$, the set $\mathcal{P}$ of integer partitions ripples with interesting patterns and relations. Now, as Alladi-Erdős point out, the prime decompositions of integers are in bijective correspondence with the set of partitions into prime parts, if we associate 1 to the empty partition. Might some number-theoretic theorems arise as images in $\mathbb{Z}^+$ (i.e. in prime partitions) of greater algebraic and set-theoretic structures in $\mathcal{P}$? In the 1970s, Andrews developed a beautiful theory of partition ideals using ideas from lattice theory, teasing the possibility of a universal algebra of partitions. Looking in a similar direction for arithmetic structures in the partitions, we show that many objects from elementary and analytic number theory are special cases of general partition-theoretic and q-series theorems: a multiplicative arithmetic of partitions that specializes to classical cases; a class of "partition zeta functions" containing $\zeta(s)$ and other Dirichlet series as well as exotic non-classical cases; partition formulas for arithmetic densities of subsets of $\mathbb{Z}^+$ such as kth-power-free integers; and other phenomena at the intersection of the additive and multiplicative branches of number theory. (Received September 14, 2017)