

1135-11-1321

**Hugh L. Montgomery\*** ([hlm@umich.edu](mailto:hlm@umich.edu)), Department of Mathematics, University of Michigan, 530 Church Street, Ann Arbor, MI 48109-1043. *New Trigonometric Extremal Problems Related to Pair Correlations*. Preliminary report.

Assume the Riemann Hypothesis (RH), and for real  $\alpha$  and  $T \geq 2$ , put

$$F(\alpha, T) = \left( \frac{T}{2\pi} \log T \right)^{-1} \sum_{\substack{0 < \gamma \leq T \\ 0 < \gamma' \leq T}} T^{i\alpha(\gamma - \gamma')} w(\gamma - \gamma')$$

where  $w(u) = 4/(4 + u^2)$ . We know that  $F$  is real valued, even, nonnegative, and that

$$F(\alpha) = (1 + o(1))T^{-2\alpha} \log T + \alpha + o(1)$$

uniformly for  $0 \leq \alpha \leq 1$ . If  $R \in L^1(\mathbb{R})$ , then

$$\sum_{\substack{0 < \gamma \leq T \\ 0 < \gamma' \leq T}} \widehat{R}\left(\frac{\gamma - \gamma'}{2\pi} \log T\right) w(\gamma - \gamma') = \left(\frac{T}{2\pi} \log T\right) \int_{\mathbb{R}} R(\alpha) F(\alpha) d\alpha.$$

This has been used to obtain results concerning the spacing of the zeta zeros. In the past it was required that  $R(\alpha) = 0$  when  $|\alpha| > 1$ . We note that this is overly severe: For many purposes it suffices to constrain the sign of  $R$  in this range. (Received September 21, 2017)