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Victor Y. Wang* (vywang@mit.edu). *On Hilbert 2-class fields and 2-towers of imaginary quadratic number fields.*

Inspired by the Odlyzko root discriminant and Golod–Shafarevich p -group bounds, Martinet (1978) asked whether an imaginary quadratic number field K/\mathbb{Q} must always have an infinite Hilbert 2-class field tower when the class group of K has 2-rank 4, or equivalently when the discriminant of K has 5 prime factors. No negative results are known. Benjamin (2001, 2002) and Sueyoshi (2004, 2009, 2010) systematically established infinite 2-towers for many K in question, by casework on the associated Rédei matrices. Others, notably Mouhib (2010), have also made progress, but still many cases remain open, especially when the the class group of K has small 4-rank.

Recently, Benjamin (2015) made partial progress on several of these open matrices when the class group of K has 4-rank 1 or 2. In this paper, we partially address many open cases when the 4-rank is 0 or 2, affirmatively answering some questions of Benjamin. We then investigate barriers to our methods and ask an extension question (of independent interest) in this direction. Finally, we suggest places where speculative refinements of Golod–Shafarevich or group classification methods might overcome the ‘near miss’ inadequacies in current methods. (Received September 06, 2015)