Ryan L Kaliszewski* (rlk72@drexel.edu), Department of Mathematics, Drexel University, 3141 Chestnut Street, Philadelphia, PA 19104, and Huilan Li. Symmetry of the rational $q, t$-Catalan numbers for $3, n$-Dyck paths.
The Catalan numbers are one of the most classical of integer sequences. The more general "(m,n)-rational Catalan numbers", count Dyck paths inscribed in an $n \times m$ integer lattice where $n$ and $m$ are coprime, have been the subject of recent attention as it has been discovered that they are connected to many classical and contemporary theories. For example, they are in bijection with a subset of elements in the Coxeter group of type-A.

The rational $q, t$-Catalans are polynomials in two parameters t and q defined by summing $q^{\operatorname{area}(\pi)} t^{\operatorname{dinv}(\pi)}$ over all Dyck paths $\pi$ in the $n \times m$ integer lattice, where $(\operatorname{area}(\pi), \operatorname{dinv}(\pi))$ is a distinguished pair of non-negative integers associated to each path $\pi$. Of particular interest is the conjecture that the rational Catalan numbers are symmetric in $q$ and $t$.

We prove the cases when $m=2$ and $m=3$ by describing the rank word of a $m, n$-Dyck path, which is an increasing list of the positive ranks of the cells in the diagram that highlights those ranks associated to cells lying above the path. The rank words uniquely determine the $n, m$-Dyck paths when $m=2$ and $m=3$ and allow construction of an involution on these paths that exchange the dinv and area statistics. (Received September 15, 2014)

