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Eisenstein integers are defined to be the set $Z[\omega] = \{a + b\omega : a, b \in \mathbb{Z}\}$ where $\omega = (-1 + i\sqrt{3})/2$. This set lies inside the set of complex numbers \mathbb{C} and they also form a commutative ring in the algebraic number field $\mathbb{Q}(\omega)$. In this paper we prove a few results related to the factor rings over the Eisenstein integers. In particular we show that the ring $Z[\omega]$ factored by an ideal generated by any element $m + n\omega$ of this ring, where $\text{g.c.d}(m, n) = 1$ is isomorphic to the ring $Z_{N(m+n\omega)}$, where N is the norm function given by $N(m + n\omega) = (m + n\omega)(m + n\bar{\omega}) = m^2 + n^2 - mn$. This result helps us quickly answer questions about the number of elements of the factor ring $Z[\omega]/\langle m + n\omega \rangle$. Then, we give a representation for the factor ring $Z[\omega]/\langle m + n\omega \rangle$ in terms of simpler rings. Finally, at the end we give a few applications to elementary number theory, more specifically we use some of the results in this paper to find all solutions of the equation $p = m^2 + n^2 - mn$ where p is a prime not congruent to 2 mod 3, and $m, n \in \mathbb{Z}^+$. (Received August 30, 2011)