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Existence, uniqueness and asymptotic phase in the carrying simplex for certain competitive maps.

X is a star-shaped neighborhood of 0 in \mathbf{R}_+^n . The vector order in any face $F \subseteq \mathbf{R}_+^n$ is denoted by \leq_F . Let $T : X \rightarrow X$ be continuous and map $X \cap F$ into itself for each open face F , including $\text{Int}(\mathbf{R}_+^n)$. A compact invariant set $\Sigma \subset X$ is a *carrying simplex* if it attracts all trajectories except the origin and meets every line through the origin in \mathbf{R}^n in a unique point.

Theorem: Assume: T is strictly sublinear, $x <_F y$ if $Tx \ll_F Ty$, and the origin is in the interior of the compact global attractor. Then there is a unique carrying simplex Σ , it is unordered, and every trajectory except the origin is asymptotic with a trajectory in Σ .

Examples with $T = (T_1, \dots, T_n) : \mathbf{R}_+^n \rightarrow \mathbf{R}_+^n$:

(1) $T_i(x) = x_i \exp(B_i - \sum_j A_{ij}x_j)$, where $B_i, A_{ij} > 0$ and $\sum_j \frac{B_i A_{ij}}{A_{ii}} < 1$

(2) $T_i(x) = \frac{C_i x_i}{1 + \sum_j A_{ij}x_j}$, where $A_{ij} > 0$ and $1 < C_i < 1 + \frac{A_{ii}}{\sum_j A_{ij}}$

(3) The Poincaré map of certain competitive periodic systems of differential equations in \mathbf{R}_+^n . (Received August 19, 2007)