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Michael Robinson* (robinm@cam.cornell.edu), 657 Rhodes Hall, Cornell University, Ithaca, NY 14850. *Rigorous asymptotic-numeric examination of the dynamics of semilinear parabolic equations.*

For nonlinear parabolic equations on unbounded domains, numerical methods can provide misleading information about solution dynamics. For instance, solutions which appear to be well-behaved may in fact blow up. Worse, if an equilibrium has a low-dimensional stable manifold, it may be overlooked by a purely numerical examination.

For concreteness, we examine the Cauchy problem for

$$\frac{\partial u(t, x)}{\partial t} = \frac{\partial^2 u(t, x)}{\partial x^2} - u^2(t, x) + \phi(x),$$

where $t > 0$, $x \in \mathbb{R}$, and ϕ is smooth and decays to zero. A two-step approach is presented, one which first finds the equilibria using a combination of asymptotic and numerical methods, and then searches locally near each equilibrium for heteroclinic orbits between the equilibria. The first step creates an equilibrium solution valid on the entire (unbounded) domain by using asymptotic methods to solve the equilibrium problem “near infinity”, and then by using a numerical solver for elliptic boundary value problems to solve “near zero”. The second step uses a numerically stable parabolic solver to start solutions near the computed equilibria. In both steps, rigorous estimates of the errors are given to prove that the dynamics are indeed correct. (Received September 19, 2007)