Jonathan E. Beagley* (beagjon@iit.edu), 1394 Cottonwood Lane, Crystal Lake, IL 60014, Eileen L. Radzwion (radzw1el@cmich.edu), 810 S. University, Mt. Pleasant, MI 48858, and Andrew M. Zimmer (azimmer@ups.edu), 3601 WSC, Tacoma, WA 98416. Minimum Semidefinite Rank of a Graph with Cut Sets of Size 2 and Graphs with msr $(G)=|G|-2$.
Given a Hermitian matrix $A \in M_{n}(\mathbb{C})$, associate a simple, undirected graph $G(A)$ where $V(G)=\{1,2, \ldots, n\}$ and $E(G)=\left\{i j \mid a_{i j} \neq 0, i \neq j\right\}$. The collection of all Hermitian matrices that share a common graph $G$ is denoted $\mathcal{H}(G)$. The problem of finding the multiplicities of the eigenvalues among the matrices in $\mathcal{H}(G)$ has received much attention recently. In this presentation we consider $\mathcal{P}(G) \subset \mathcal{H}(G)$ where $\mathcal{P}(G)$ is the set of all positive semidefinite matrices corresponding to $G$. The minimum semidefinite rank of $G$, denoted $\operatorname{msr}(G)$, is defined to be the minimum rank among all matrices in $\mathcal{P}(G)$.

To each vertex $i$ in a graph $G$ we associate a vector $\overrightarrow{v_{i}} \in \mathbb{C}^{m}$ such that for $i \neq j, i j \in E(G)$ if and only if $\left\langle\overrightarrow{v_{i}}, \overrightarrow{v_{j}}\right\rangle \neq 0$. The set of vectors $\left\{\overrightarrow{v_{1}}, \overrightarrow{v_{2}}, \ldots, \overrightarrow{v_{n}}\right\}$ is called a vector representation of $G$. Using vector representations we present results on finding the $\operatorname{msr}(G)$ when $G$ is written as a "vertex sum" of two graphs $G_{1}$ and $G_{2}$ that share a cut set of at most two vertices. Lastly, a classification of all graphs with $\operatorname{msr}(G)=|G|-2$ will be discussed. (Received July 25, 2007)

