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Dragos Ghioca, Thomas J Tucker and Michael E Zieve* (zieve@math.rutgers.edu),
Center for Communications Research, 805 Bunn Drive, Princeton, NJ 08540. *Intersections of
polynomial orbits.*

Pick nonlinear $f, g \in \mathbb{C}[x]$, and arbitrary $x_0, y_0 \in \mathbb{C}$. I will discuss the following result: if the orbits $\{x_0, f(x_0), f(f(x_0)), \dots\}$ and $\{y_0, g(y_0), g(g(y_0)), \dots\}$ have infinite intersection, then f and g have a common iterate.

The main ingredients in the proof are Siegel's theorem on integral points on curves, an analogue for preperiodic points of Silverman's specialization theorem, Ritt's characterization of polynomials with a common iterate, inequalities involving canonical heights, a result of Bilu-Tichy on diophantine equations with infinitely many solutions, and various new results about polynomial decomposition, of which the most difficult is: if the k -th iterate of f can be written as the composition $u \circ v$ with $u, v \in \mathbb{C}[x]$, then one of the following holds: (1) for some $h \in \mathbb{C}[x]$ we have either $u = f \circ h$ or $v = h \circ f$; (2) $k < \log_2(\deg f)$; or (3) for some linear $\ell \in \mathbb{C}[x]$ we have either $f = \ell \circ x^n \circ \ell^{-1}$ or $f = \ell \circ \epsilon T_n \circ \ell^{-1}$, where $\epsilon \in \{1, -1\}$ and T_n is the Chebychev polynomial, defined by $T_n(\cos \theta) = \cos n\theta$. (Received July 17, 2007)