1035-11-986   **Eric S. Rowland*** (`erowland@math.rutgers.edu`), Department of Mathematics, Rutgers
University, Piscataway, NJ 08854. *A simple prime-generating recurrence.*

In 2003 it was conjectured that the somewhat unusual recurrence $f(n) = f(n-1) + \gcd(n, f(n-1))$ with, for example, $f(1) = 7$, has a sequence of differences $f(n) - f(n-1) = \gcd(n, f(n-1))$ that is comprised entirely of 1s and primes:
1, 1, 1, 5, 3, 1, 1, 1, 1, 11, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 23, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 47, 3, 1, 5, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 101, 3, 1, 1, 7, 1, 1, 1, 1, 11, 3, 1, 1, 1, 1, 1, 13, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 233, 3, 1, 1, . . .

While $f(n)$ clearly has something to do with factorization (due to the gcd), it is not obvious why $\gcd(n, f(n-1))$ should never be composite. There is, however, sufficient structure to allow a proof, making this recurrence a rare "naturally occurring" generator of primes. (Received September 18, 2007)