1035-05-355 **Jeffrey O. Shallit** (shallit@graceland.uwaterloo.ca), School of Computer Science, University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada, and **Zhi Xu*** (z5xu@cs.uwaterloo.ca), School of Computer Science, University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada. *The Frobenius Problem in a Free Monoid.*

The classical Frobenius problem is to compute the largest integer g not representable as non-negative integer linear combination of $x_1, x_2, \ldots, x_k$, where $x_1, x_2, \ldots, x_k$ are positive integers with $\gcd(x_1, x_2, \ldots, x_k) = 1$. We generalize this problem to the non-commutative setting of a free monoid, where $S = \{u_1, u_2, \ldots, u_k\}$ is a set of words over a finite alphabet Σ such that S^*, the set of all words factorizable into elements of S, is co-finite. We use techniques from automata theory and formal language theory to discuss the length of the longest word not in S^*.

Unlike the commutative case, where the bound on $g(x_1, x_2, \ldots, x_k)$ is quadratic, we are able to prove that the length of the longest word not in S^* is bounded above by

$$\frac{2(2^n|\Sigma|^n - 1)}{2|\Sigma| - 1},$$

where $n = \max_{1 \le i \le k} |u_i|$. Furthermore, we are able to show a tight (exponential) lower bound for the worst case of the form

$$g(m, m|\Sigma|^{n-m} + n - m)$$

in the case where $S \subseteq \Sigma^m \cup \Sigma^n$, where $0 < m < n < 2m$.

We obtain upper and lower bounds for other generalizations of the Frobenius problem, such as the state complexity of S^*, and we also obtain results on the total number of words not in S^*, generalizing an 1884 result of Sylvester. (Received September 04, 2007)