

1035-00-9

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Bat 425, Orsay, Cedex 91905. *Random conformally invariant pictures (Part I)*.

In general, when one observes a system where many random inputs come together via finitely many quantities (one sums up the behavior of the system via a number for instance), then this quantity tends to become deterministic when the number of inputs gets very large. The randomness is in some way averaged out, and one sees a non-random system on large scale. Sometimes however, and this is in particular the case for physical systems at the precise temperature where a “phase transition” occurs, it happens that the large-scale behavior remains random.

Physicists have told us for more than twenty years that something special is going on for two-dimensional systems that enable us to describe mathematically this phase transition and the corresponding random systems. The notion of conformal invariance lies at the root of Conformal Field Theory that had been able to predict various features of these two-dimensional phenomena.

In recent years, mathematicians have succeeded in understanding and proving these predictions. An important step was the definition in 1999 of the Stochastic Loewner Evolution by Oded Schramm.

The goal of these lectures will be to describe various general ideas on this topic (due to many people including Greg Lawler, Oded Schramm, Scott Sheffield, Stas Smirnov and myself), trying to make use of the two-dimensional screen to show two-dimensional pictures.

This first lecture will be a general overview – based on specific examples – on how the combination of a large number of random inputs can generate deterministic or random behavior on large scale. Using the example of planar percolation, we shall in particular discuss the notion of phase transition and also illustrate the meaning of stability of random macroscopic events with respect to perturbations.

(Received May 01, 2007)