I: Machine Assisted Proof

For centuries, mathematicians have relied on computers to perform calculations, to suggest conjectures, and as components of mathematical proofs. In the light of more modern tools, such as interactive theorem provers, machine learning algorithms, and generative AI, we are beginning to see machines used in more creative and substantive ways in our work. In this talk we survey some historical and recent developments, and speculate on the future roles of machine assistance in mathematics.

II: Translational tilings of Euclidean space

Translational tilings of Euclidean space are a partition of Euclidean space (up to null sets) by translates of one or more tiles. For instance, the famous Penrose tilings are a translational tiling by a finite number of triangles which can only tile the plane nonperiodically. One of the outstanding problems in this field is the "periodic tiling conjecture" –if a tile can tile space by translations, must there exist a way to tile space periodically with this tile? Recently, Rachel Greenfeld and I were able to disprove this conjecture in high dimensions, by encoding a certain "Sudoku puzzle" as a tiling problem. We survey this result and other recent progress in this talk.

III: Correlations of multiplicative functions

The Liouville function assigns a sign to each natural number: +1 if the number is the product of an even number of primes, and -1 if the number is the product of an odd number of primes. A famous conjecture of Chowla asserts consecutive values of the Liouville function are asymptotically uncorrelated to each other; this can be viewed as a simplified analog of the twin prime conjecture. This conjecture is still open today, but significant progress has been made toward it; while we still cannot quite detect cancellations in the Liouville function at small enough scales to imply the Chowla conjecture, we have been able to see such cancellations in slightly larger scales. Nevertheless, there still appears to be a key ingredient missing to complete the story. We survey some of the recent developments in this area.

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AMS Colloquium Lecture I, Wednesday January 3, 2024, 1:00 p.m.–2:00 p.m. Moscone, Room 207

AMS Colloquium Lecture II, Thursday January 4, 2024, 1:00 p.m.–2:00 p.m. Moscone, Room 207

AMS Colloquium Lecture III, Friday January 5, 2024, 1:00 p.m.–2:00 p.m. Moscone, Room 207

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