THE PH.D. DEGREE AND MATHEMATICAL RESEARCH*

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Recommendations regarding the training and utilization of advanced students of mathematics must be based on specific information concerning the present situation and on a knowledge of how the past has contributed to its upbuilding. No group of persons can be entirely certain that sound deductions are possible from data as incomplete as those now available. However, queries often raised regarding quality of personnel and regarding supply and demand can be answered with considerable assurance. In so complicated a problem any light that can be shed is undoubtedly welcome, and it is proposed in this report to set forth a variety of facts and to venture partial and tentative answers to certain questions.

We propose questions such as the following: How many doctor's degrees have been conferred on American mathematicians here in the United States and Canada? How many in Europe? What proportion of the present teachers of mathematics in colleges and universities have such degrees? Is there a sufficient number of competent students of mathematics now being enlisted and subjected to proper training by our graduate schools? What percentage of the Ph.D.'s have published considerable research? Is the record of publication improving with the newer crop of Ph.D.'s? What universities have the distinction of the largest average amount of publication by those to whom they have granted degrees? How does the record of the National Research Fellows stand?

Beginning with 1900, the Bulletin of the American Mathematical Society has made a practice of publishing an annual list of the doctor's degrees conferred in mathematics. In addition, each university has, on request, been kind enough recently to furnish lists of all doctor's degrees in mathematics conferred since the beginning of its graduate work and including the year 1934. From these sources† accurate data have been assembled. The corresponding information regarding Americans and Canadians taking degrees abroad is not so complete.

Each year, beginning with 1890, the Bulletin of the American Mathematical Society has printed an *Annual List of Papers Read Before the Society and Subsequently Published*. These lists have been drawn on for the period 1890–1933, and other sources for the period before 1890. It has been possible to make a complete tabulation for the sixty years ending with 1933,‡ giving the number of such papers and the number of pages printed. It should be pointed out, however, that papers of American authorship not presented to the Society are

* A report to the Commission on the Training and Utilization of Advanced Students of Mathematics presented at its session on December 31, 1935.

† Beginning with 1934, a list of doctors and thesis titles in all fields of advanced study is being published annually by H. W. Wilson Company under the title *Doctoral Dissertations Accepted by American Universities* compiled for the National Research Council and the American Council of Learned Societies by the Association of Research Libraries.

‡ For the National Research Fellows the tabulation has been extended to include 1934.

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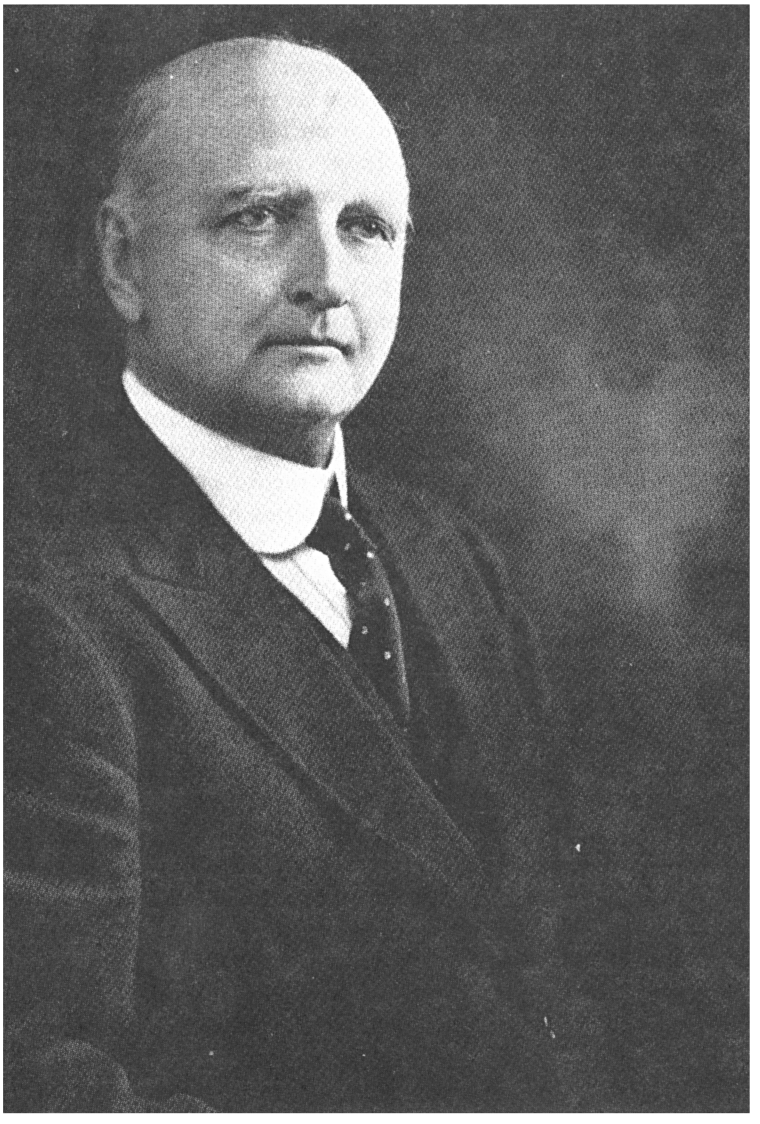
Note: Although subsequent research has led to changes in the statistics that Richardson displays, so that his numbers should no longer be used without verification, the paper remains a model of the type of study that periodically increases our awareness of the status of the field. **(The Editors)**



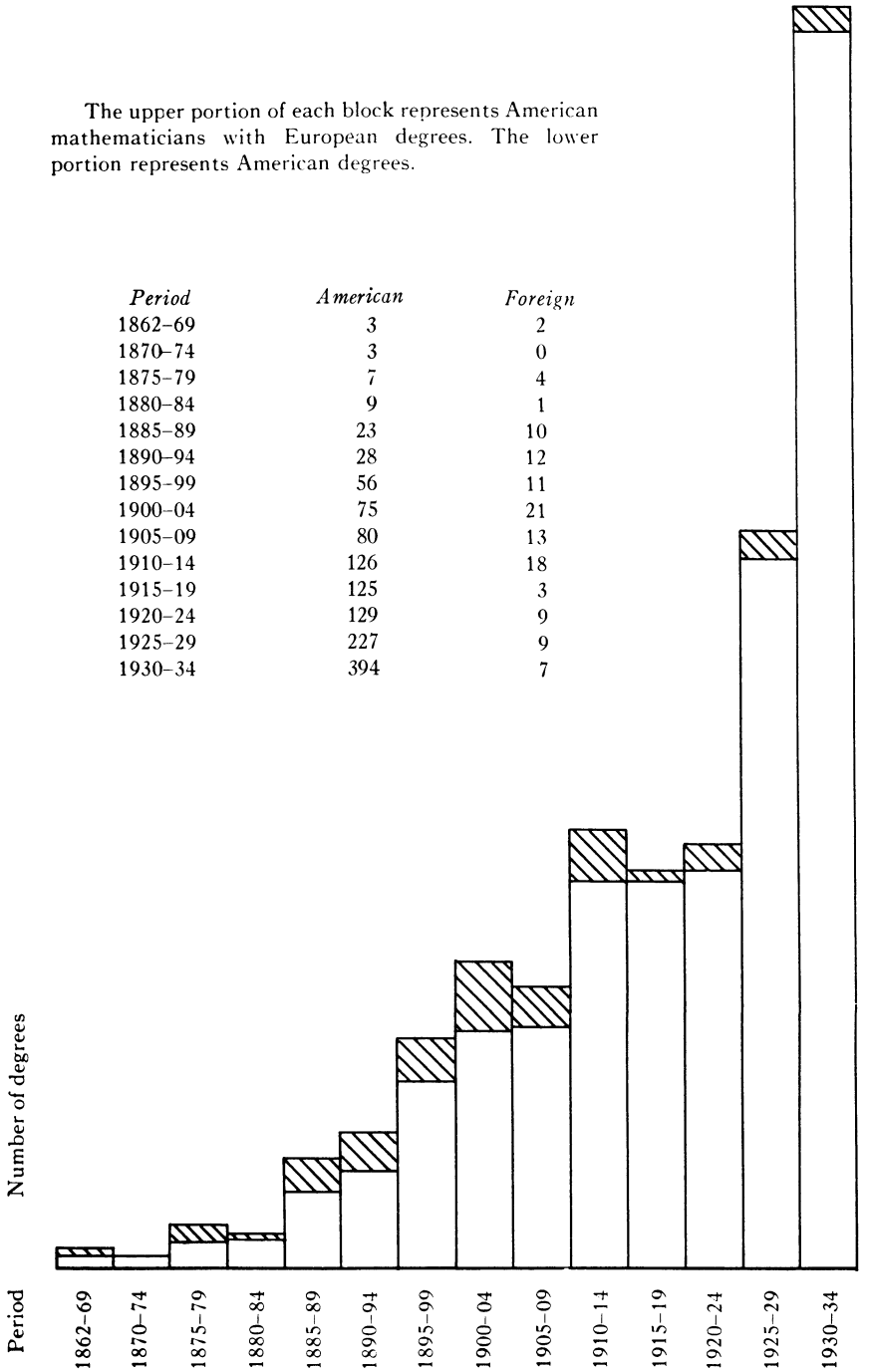
R. G. D. Richardson

GRAPH I

DISTRIBUTION OF DEGREES CONFERRED IN MATHEMATICS 1862–1934

The upper portion of each block represents American mathematicians with European degrees. The lower portion represents American degrees.

Period	*American*	*Foreign*
1862–69	3	2
1870–74	3	0
1875–79	7	4
1880–84	9	1
1885–89	23	10
1890–94	28	12
1895–99	56	11
1900–04	75	21
1905–09	80	13
1910–14	126	18
1915–19	125	3
1920–24	129	9
1925–29	227	9
1930–34	394	7



omitted from the tabulations; and this is true whether these papers appeared in American or foreign journals. Some mathematicians, especially among the more eminent, publish papers not read before the Society; but a study of the total publication of mathematical research for a typical year (1932) shows that only about 22% of such publication (1,158 pages out of a total of 5,290) is not included in our data. The data assembled seem, therefore, fairly adequate for the purpose in hand.

For each person it has been possible to compute an index of publication which is the annual average, measured in pages, of printed research during the period subsequent to receiving the degree, but in no case exceeding twenty-five years. The average of the indices for the group of doctors beginning with 1862 is 4.73, which then establishes a norm by which comparisons can be made. Otherwise stated, a doctor publishes yearly, on the average, 4.73 pages of research which has been presented to the Society. (If the year 1932 referred to above is a fair sample, this figure would be increased to 6 pages by including papers not presented to the Society. And it should be expressly understood that the list includes only papers published in research journals, and that separate monographs and books, which are so important in the development of research, are excluded from the count.) Reckoning eight dollars as the cost of printing a page, the average annual cost of publication per doctor is approximately fifty dollars, a relatively small item in the cost of production of research.

As a rough check on this figure of 4.73 pages, we note that the American Journal, Annals, Bulletin, and Transactions print about 3,500 pages of research annually and that other journals add perhaps 1,600 pages more. This makes a total of 5,100 pages as against approximately 1,200 doctors.

Number of Ph.D.'s Conferred in the United States and Canada

A careful investigation indicates that the number of Ph.D.'s in mathematics conferred by institutions in the United States and Canada during the period 1862–1934 is 1,286, 168 of which were conferred on women. Graph I exhibits the number of degrees conferred during five-year periods, this number following in general an exponential curve, though interrupted during the world war. The totals by years for the period 1930–34 are 84, 79, 69, 71, 92, respectively.

The figures naturally vary somewhat with the inclusion or exclusion of degrees taken in applied fields such as mathematical physics, mathematical astronomy, mechanics, theory of statistics, etc.; but they are accurate enough for our purposes.

Besides this number of American degrees, the information available indicates that there have been, during the period 1862–1930, 114 degrees conferred by foreign universities on mathematicians* who have been active in America.†

* The University of Göttingen accounts for 34 (Henry Blumberg, Maxime Bôcher, Oskar Bolza, Ann L. Bosworth, W. D. Cairns, A. R. Crathorne, H. B. Curry, Edgar Dehn, T. M. Focke, D. C. Gillespie, Charles Haseman, M. W. Haskell, E. R. Hedrick, W. A. Hurwitz, Dunham Jackson, O. D. Kellogg, A. J. Kempner, S. D. Killam, Luise Lange, Heinrich Maschke, Max Mason,

Included in this list are 40 mathematicians of European birth who subsequently took up residence in America and thoroughly established themselves by residence of several years.

Yale University, the first institution in America to confer the Ph.D. degree in course (1861), awarded the degree in mathematics in 1862 to J. H. Worrall. William Watson, later professor at Harvard University, received from Jena in 1862 the first foreign degree of which we have record. The earliest degrees conferred on women were granted to Winifred H. Edgerton by Columbia University in 1886 and to Charlotte A. Scott by the University of London in 1885.

As will be noted from Table I, more than one-sixth of the 1,286 degrees conferred in America have been awarded by the University of Chicago alone. Six institutions—Chicago, together with Cornell, Harvard,* Illinois, Johns Hopkins, and Yale—are responsible for more than half. Of the remaining 53 institutions, there have been 7 which have each conferred 25 or more degrees (California, Clark, Columbia, Michigan, Pennsylvania, Princeton, and Wisconsin) and 14 that have each conferred from 10 to 24 degrees (Brown, Bryn Mawr, Catholic, Cincinnati, Indiana, State University of Iowa, Massachusetts Institute, Minnesota, Missouri, Ohio State, Pittsburgh, Rice, Texas, and Virginia). The remaining 94 degrees were given by 32 institutions. During the past ten years only one-half the universities on this list (29 out of 59) have conferred five or more degrees and can thus be considered an important present factor. In the five years 1930–34, Chicago maintains its lead in the number of degrees conferred with 52, while Michigan is second with 35, followed by Cornell and Harvard each with 28.

C. A. Noble, L. W. Reid, W. B. Smith, Virgil Snyder, Elijah Swift, J. H. Tanner, E. J. Townsend, E. B. Van Vleck, Paul Wernicke, W. D. A. Westfall, H. S. White, Mary F. Winston, F. S. Woods); the others are distributed as follows: Berlin 4 (Harris Hancock, Ludwik Silberstein, A. G. Webster, E. J. Wilczynski), Bonn 2 (J. L. Coolidge, W. C. Graustein), Bordeaux 1 (P. L. Saurel), Cambridge 4 (E. W. Brown, R. C. Maclaurin, Frank Morley, H. M. Tory), Charlottenburg 1 (E. P. Wigner), Dublin 2 (James McMahon, J. L. Synge), Edinburgh 2 (Alexander Macfarlane, J. H. M. Wedderburn), Erlangen 4 (Henry Benner, W. F. Osgood, E. D. Roe, H. W. Tyler), Fribourg 1 (R. F. Schnepp), Ghent 1 (N. A. Court), Greifswald 1 (F. J. Dohmen), Heidelberg 2 (R. R. Fleet, T. E. Hart), Jena 1 (William Watson), Kazan 1 (G. Y. Rainich), Kiel 1 (Arthur Schultze), Königsberg 1 (J. B. Chittenden), Leiden 1 (D. J. Struik), Leipzig 9 (H. F. Blichfeldt, C. L. Bouton, H. B. Fine, A. G. Hall, J. M. Page, B. O. Peirce, D. A. Rothrock, W. E. Story, A. F. Wintner), London 1 (Charlotte A. Scott), Munich 9 (G. N. Armstrong, C. H. Ashton, P. S. Epstein, L. A. Howland, W. W. Küstermann, H. W. March, G. W. Myers, W. S. Seidel, Edwin R. Smith), Oslo 1 (Oystein Ore), Oxford 2 (W. R. Burwell, F. V. Morley), Paris 2 (Pierre Boutroux, Jacques Chapelon), Rome 1 (Oscar Zariski), St. Anthony 1 (J. A. Caparó y Peréz), St. Petersburg 4 (Alexander Chessin, J. A. Shohat, J. D. Tamarkin, J. V. Uspensky), Salzburg 1 (Eberhard Hopf), Stockholm 1 (Einar Hille), Strasbourg 5 (R. C. Archibald, J. W. Bradshaw, Myrtie Collier, E. V. Huntington, L. C. Karpinski), Szeged 1 (Tibor Radó), Tübingen 2 (S. C. Davisson, J. E. Manchester), Upsala 1 (T. H. Gronwall), Utrecht 1 (Hermance Mullemeister), Vienna 3 (F. W. Doermann, J. L. Gibson, James Pierpont), and Zurich 5 (W. H. Butts, S. R. Epsteen, Lulu Hofmann, William Marshall, A. B. Pierce).

† When persons have taken two doctors' degrees, the first only is listed.

* Throughout this paper, statistics for Harvard include Radcliffe.

TABLE I
NUMBER OF PH.D. DEGREES IN MATHEMATICS CONFERRED BY AMERICAN UNIVERSITIES

Institution	1862–69	70–79	80–89	90–94	95–99	00–04	05–09	10–14	15–19	20–24	25–29	30–34	Total
Boston U.								3					3
Brown											1	13	14
Bryn Mawr				1	1	2		1	1	2	3	2	13
Calif. Inst. of Tech.											5	3	8
California						2	1	5	7	8	11	11	45
Catholic									3		3	9	15
Chicago					4	15	20	25	31	41	49	52	237
Cincinnati								1		1	1	7	10
Clark				4	7	7	2	4	2				26
U. of Colorado							1					1	2
Columbia U.			4	1	3	6	5	11	10	5	6	11	62
Cornell U.		1	3	1	1	7	8	8	8	8	16	28	89
Cumberland				1									1
Dartmouth		1											1
Duke												3	3
Fordham											2		2
Geo. Washington				1								2	3
Harvard and Radcliffe		2	3		5	5	7	11	15	11	16	28	103
Haverford						1							1
Illinois						1		6	8	10	25	23	73
Indiana								2	3		1	5	11
Iowa State												1	1
State U. of Iowa										3	5	9	17
Johns Hopkins		2	12	7	9	8	10	14	7	8	15	11	103
Kansas					1			2			2	1	6
Kentucky												1	1
Lafayette			1	1									2
Marquette												2	2
Mass. Inst. of Tech.										1	4	12	17
Michigan								4	5	3	8	35	55
Minnesota										2	4	4	10
Missouri								1	1	2	4	2	10
Moravian									1				1
Nebraska					2	1					1		4
New York									1		2	2	5
U. of N. Carolina						1						1	2
Ohio State U.												16	16
Ohio Wesleyan				1									1
Otterbein				1									1
U. of Pennsylvania				2	7	3	6	5	7	2	9	11	52
Pittsburgh											1	12	13
Princeton			2	1		1	2	8	3	9	4	18	48
Purdue				1									1
Rensselaer										1		2	3
Rice									1	1	6	5	13
St. Louis U.												2	2
Stanford						1		1			2	3	7
Syracuse U.			1	1		1		2	1	3			9
U. of Texas									1	1	5	4	11
U. of Toronto									1		1	6	8
Tulane				1									1
Vanderbilt			1				1						2
U. of Virginia			1		3	1	3		1			2	11
Washington U.				1								2	3
U. of Washington											2	3	5
West Virginia U.												2	2
Wisconsin					2	1	2	1	3	5	5	20	39
Wooster			1										1
Yale	3	4	3	2	11	11	12	11	4	2	8	8	79
Totals	3	10	32	28	56	75	80	126	125	129	227	395	1286

In 1933 a committee of the American Council on Education made a study of institutions conferring Ph.D. degrees. As the result of a ballot in each field, a group of institutions was designated as approved, and from that list the most distinguished were singled out by stars. Of the 395 degrees in mathematics granted in 1930–34, 157 were conferred by institutions starred in this field, 187 by other institutions approved in the field, and 51 (about 12%) by institutions not on the approved list.

Of the 32 institutions now members of the Association of American Universities, 28 have conferred the degree in mathematics during the five years 1930–34, the total being 348, or 88% of all such degrees conferred.

Foreign Influence on American Mathematics

Grateful acknowledgment should be made of America's tremendous debt to foreign universities. The original inspiration for work in a large number of the fields now cultivated here had its source on the other side of the Atlantic, especially in Germany. As will be noted from Graph I, the number of foreign degrees during the period 1885–1914 was a considerable proportion of the whole; not only is this group important numerically but, as will be seen later in this discussion, it is significant scientifically. During the period in question, many of the ablest of our students went abroad to obtain degrees. Harvard in particular followed a very generous policy in awarding fellowships to persons wishing to take degrees in Europe. Up to and including 1913, Harvard sent abroad 26 mathematical fellows, of whom 16 took European degrees.*

Beginning with 1914, however, the number of Americans taking their degrees abroad declined rapidly. By that time the custom had been established of taking degrees in this country, even when the students wished to pursue further study abroad. The number of American mathematicians attending foreign universities in recent years is by no means so great as it was a quarter of a century ago, but scholars in this country are still indebted to universities in Europe for much stimulation. There seems to be no doubt that by our connection with foreign countries we have avoided and are avoiding the inbreeding which is apt to lead to sterility.

During the past decade the international fellowships, supported by the Rockefeller Foundation, have been of great importance in establishing contacts between American and European mathematicians. Other recent factors are the importation of some European scholars through the founding of the Institute for Advanced Study, and the recent infiltration of other displaced German scholars into our mathematical group. Besides the 40 foreign-born mathematicians referred to above as having entered the United States and Canada before 1930, there have been approximately 20 mathematicians who have more recently transferred to these countries from Europe, thus further raising the volume of our research activity.

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* This includes B. O. Peirce and A. G. Webster whose fields bordered on physics.

While the tide of students venturing forth to Europe has abated since the world war, the number of foreign students arriving on our shores has rapidly increased, drawn partly by the fame of our leaders in scholarship and partly by the hope that a career might open for them here. Leadership in many fields is definitely passing to America, and the tides of students to and from Europe need to be regulated by an informed opinion with a view to best serving the cause of American scholarship.

Number of College Teachers of Mathematics

Data collected in the autumn of 1935, based on information furnished by the institutions themselves, indicate, as tabulated in Table II, that the number of persons teaching mathematics in colleges, universities, junior colleges, and degree-granting normal colleges in the United States (with its outlying possessions) and Canada is approximately 4,500. This includes some persons who are teaching descriptive geometry, mechanics, and methods in mathematics, as well as some who are teaching part time or who are largely in administrative work or who are emeriti; but on a conservative estimate, 4,000 persons are actually engaged full time in the teaching of mathematics of college freshman grade or higher. Similar figures were collected in 1932, and there seems to have been a considerable increase in the interim, due chiefly to the growth of junior colleges.

Table II is a statistical study by states of the number of teachers of mathematics in junior colleges, teachers colleges, and other colleges and universities, of the number of men and women teachers, of the number of teachers holding doctor's degrees, and of the number of those who are members of the American Mathematical Society or of the Mathematical Association of America or of both. The best information obtainable indicates that probably somewhat less than 1,300 of the present teachers of mathematics have the Ph.D. degree! Many of the 1,400* listed as having obtained degrees in America or abroad are deceased or have entered fields of work such as government service, banking, or industry. It should be remarked also that mathematics has furnished more than its share of administrative officers to colleges and universities. There have been some doctors who have drifted out of mathematics into other fields of science, and probably more who have correspondingly drifted in.

Of the 4,444 teachers of mathematics listed, 1,292, or 29%, hold the degree of Ph.D., while slightly less (1,263) are members of the American Mathematical Society and slightly more (1,333) are members of the Mathematical Association. In the Summary at the end of Table II the states have been grouped by sections of the country, and we note that 35% of the teachers in the northeast section from Illinois to Maine hold the doctor's degree. Only about 22% of those in the south central states from Kentucky to Texas hold the doctor's degree. In the remainder of the country about 26% hold that degree.

* This figure does not include the 1935 crop of Ph.D.'s, most of whom are doubtless included in Table II.

TABLE II

STATISTICS REGARDING TEACHERS OF MATHEMATICS IN AMERICAN AND CANADIAN COLLEGES AND UNIVERSITIES, 1935

	Number of Teachers						Holders of Ph.D.		Membership in					
	(a)	(b)	(c)	(d)	(e)	(f)	No.	%	A.M.S.	%	M.A.A.	%	Both	%
Alabama	66	5	11	50	48	18	13	20	7	11	13	20	5	7
Alaska	3	3	—	—	3	0	0	0	0	0	0	0	0	0
Arizona	10	2	2	6	9	1	6	60	4	40	5	50	3	30
Arkansas	44	9	5	30	34	10	13	29	9	20	5	11	3	7
California	262	126	17	119	231	31	64	24	60	23	55	21	40	15
Canal Zone	2	2	—	—	2	0	1	50	0	0	0	0	0	0
Colorado	60	6	10	44	48	12	17	28	14	23	23	38	13	22
Connecticut	45	7	3	35	38	7	27	60	26	58	22	49	21	47
Delaware	7	—	—	7	6	1	2	28	5	71	4	57	3	43
Dist. of Columbia	48	13	8	27	30	18	18	37	16	33	11	23	8	17
Florida	38	5	—	33	37	1	12	31	10	26	13	34	9	24
Georgia	84	23	2	59	70	14	20	24	16	19	23	27	9	11
Hawaiian Islands	5	—	—	5	4	1	0	0	1	20	0	0	0	0
Idaho	19	9	—	10	19	0	3	16	3	16	1	5	1	5
Illinois	222	41	16	165	183	39	87	39	81	36	80	36	53	24
Indiana	96	4	8	84	87	9	33	34	25	26	42	44	15	16
Iowa	128	42	4	82	104	24	34	26	33	26	38	30	22	17
Kansas	92	24	10	58	71	21	22	24	25	27	41	44	21	23
Kentucky	77	22	19	36	56	21	19	24	14	18	28	36	12	15
Louisiana	49	7	4	38	37	12	11	22	9	18	21	43	8	16
Maine	25	2	1	22	23	2	6	24	6	24	10	40	5	20
Maryland	92	3	2	87	80	12	29	31	28	30	35	38	20	22
Massachusetts	170	14	16	140	139	31	53	31	66	39	43	25	38	22
Michigan	157	27	15	115	144	13	49	31	44	28	52	33	32	20
Minnesota	83	19	10	54	65	18	26	31	16	19	31	37	13	16
Mississippi	52	22	5	25	37	15	8	15	7	13	12	23	5	10
Missouri	122	31	21	70	97	25	30	24	27	22	29	24	12	10
Montana	19	2	1	16	18	1	4	21	7	37	5	26	4	21
Nebraska	55	5	10	40	45	10	14	25	13	24	19	34	12	22
Nevada	3	—	—	3	2	1	1	33	2	67	1	33	1	33
New Hampshire	28	1	3	24	26	2	13	46	11	39	11	39	9	32
New Jersey	107	17	7	83	102	5	35	33	44	41	29	27	24	22
New Mexico	20	3	4	13	19	1	6	30	8	40	7	35	5	25
New York	402	9	8	385	341	61	143	35	179	44	139	34	112	28
North Carolina	124	30	5	89	97	27	28	23	26	21	27	22	13	10
North Dakota	30	5	10	15	25	5	0	0	1	3	6	20	1	3
Ohio	195	4	5	186	167	28	74	38	67	34	90	46	52	27
Oklahoma	100	32	20	48	84	16	13	13	10	10	27	27	8	8
Oregon	31	4	—	27	26	5	6	19	7	22	9	29	5	16
Pennsylvania	283	12	28	243	239	44	106	37	110	39	104	37	67	24
Philippine Islands	38	—	—	38	31	7	4	10	0	0	0	0	0	0
Porto Rico	4	—	—	4	4	0	3	75	4	100	2	50	2	50
Rhode Island	32	1	2	29	29	3	13	41	19	59	14	44	13	41
South Carolina	50	3	3	44	40	10	7	14	5	10	6	12	2	4
South Dakota	27	5	2	20	24	3	6	22	7	26	6	22	3	11
Tennessee	82	10	12	60	69	13	21	26	9	11	14	17	4	5
Texas	202	62	18	122	169	33	47	23	42	21	55	27	27	13
Utah	28	13	—	15	26	2	6	21	6	21	4	14	3	11
Vermont	26	2	—	24	23	3	4	15	4	15	5	19	3	11
Virginia	98	20	6	72	74	24	23	23	26	26	29	29	21	21
Washington	47	10	5	32	44	3	19	40	18	38	13	28	11	23
West Virginia	40	6	9	25	34	6	17	42	9	22	7	17	6	15

TABLE II—*Continued*

Wisconsin	108	22	24	62	92	16	28	26	38	35	34	31	27	25
Wyoming	5	—	—	5	4	1	1	20	0	0	4	80	0	0
Canada	202	—	2	200	194	8	47	23	39	19	29	14	22	11
	4444	746	373	3325	3750	694	1292	29	1263	28	1333	30	828	19

SUMMARY

Geographic Division	Number of Teachers						Holders of Ph.D.		Membership in					
	(a)	(b)	(c)	(d)	(e)	(f)	No.	%	A.M.S.	%	M.A.A.	%	Both	%
New England	326	27	25	274	278	48	116	36	132	40	105	32	89	27
Middle Atlantic	792	38	43	711	682	110	284	36	333	42	272	34	203	26
East North Central	778	98	68	612	673	105	271	35	255	33	298	38	179	23
West North Central	537	131	67	339	431	106	132	25	122	23	170	32	84	16
South Atlantic	581	103	35	443	468	113	156	27	141	24	155	27	91	16
East South Central	277	59	47	171	210	67	61	22	37	13	67	24	26	9
West South Central	395	110	47	238	324	71	84	21	70	18	108	27	46	12
Mountain	164	35	17	112	145	19	44	27	44	27	50	30	30	18
Pacific	340	140	22	178	301	39	89	26	85	25	77	23	56	16
Territories	52	5	—	47	44	8	8	15	5	10	2	4	2	4
Canada	202	—	2	200	194	8	47	23	39	19	29	14	22	11

(a) Total; (b) Junior colleges; (c) Teachers colleges; (d) Other colleges and universities (e) Men; (f) Women.

New York State has the largest number of mathematics teachers of college grade, followed by Pennsylvania, California, Illinois, and Texas. Arizona and Connecticut are the states with the highest percentage of doctors on their faculties, with New Hampshire, West Virginia, Rhode Island, Washington, and Illinois following; at the other end of this scale is North Dakota with no doctors listed.

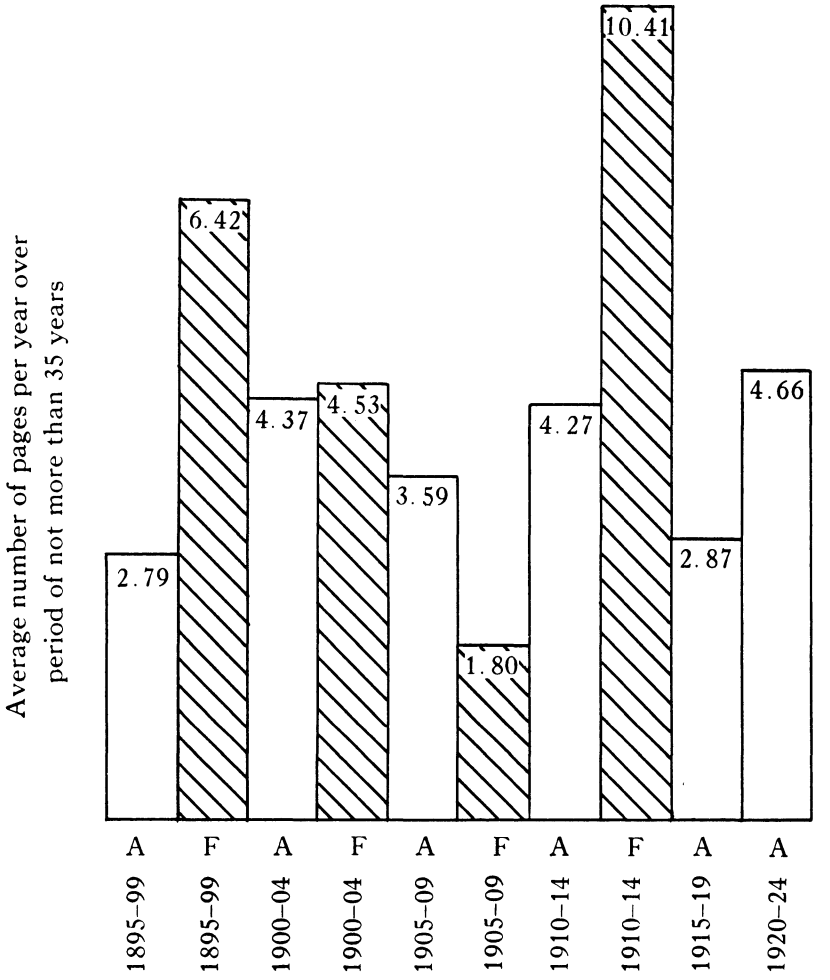
Whereas for all institutions above the rank of junior colleges (universities, colleges, and degree-granting teachers colleges) the proportion of persons with a doctor's degree is 33%, in the group of 285 colleges and universities on the approved list of the Association of American Universities it is 44%.

The decline of growth in the degree-granting institutions has been to a considerable extent offset by the establishment of emergency institutions of college freshman grade or of junior college grade. While the number of college teaching positions in mathematics probably doubled during the period 1918–1930, the unprecedented year-by-year increase in the college student population on which this increase was based was slackening rapidly even before 1929, and it seems improbable that, even if a period of economic stress had not ensued, there would have been many more academic students in the colleges of the country than there are at present. The decided drop in the number of additional appointments to the staffs of degree-granting institutions appears to have been inevitable. Without doubt the unemployment problem for college teachers is aggravated by the financial depression, but in main outlines it might have been foreseen by competent executives.

It is probably true that the number of persons attending institutions of grade beyond the high school will still increase greatly. Registration in junior colleges is increasing by leaps and bounds. The situation seems predictable, and it may be sufficiently accurate to estimate at 4,500–5,000 the number of professionally trained mathematicians who will be employed in college teaching during the next decade.

GRAPH II

COMPARISON OF PRODUCTIVITY FOR PERSONS TAKING AMERICAN AND FOREIGN DEGREES IN DIFFERENT PERIODS



Of the 60,000 persons employed in teaching all subjects in the junior colleges, colleges, and universities (not including professional schools), about 7.5% are in the departments of mathematics. In this group of institutions there are approximately 800,000 students, or about one teacher of mathematics to 175

students. In the junior colleges alone, there are approximately 6,000 teachers, of whom 12.5% are in mathematics; the total enrolment is approximately 110,000, or one teacher of mathematics to 150 students.

If we consider twenty-five years as an average period of service, it would seem that, as soon as the situation becomes a little more normal, there will be need for at least 175 new persons to be added to the staffs each year. We have arrived at the point where the universities are granting about half that number of Ph.D. degrees annually. Less than one-third of the present staffs have such degrees, and, if one-half the new appointments are made from those holding the doctorate, the situation will probably be as satisfactory as we can hope for at present. The standards in this regard need elevation, and doubtless the next decade or two will see rapid advancement. There are many college teachers with pitifully meager preparation; the institutions must look forward to their gradual replacement by well-trained men and women.

Proportion of Ph.D.'s Publishing Research Papers

It goes without saying that the number of papers and the number of pages printed is not an adequate criterion for measuring the influence of a person on mathematical thought. Often the ideas of a scholar appear in papers published

TABLE III

ANALYSIS OF NUMBERS OF PAPERS PUBLISHED BY AMERICAN MATHEMATICIANS

	Persons taking degrees 1862–1933		Persons taking degrees 1895–1924	
	Number	%	Number	%
No papers	549	46	232	39
1 paper	227	19	109	18
2 papers	100	8	58	10
3–5 papers	131	11	66	11
6–10 papers	70	6	41	7
11–20 papers	69	6	50	9
21–30 papers	20	2	17	3
More than 30 papers	22	2	17	3
Total	1188	100	590	100

by his pupils or colleagues. But a study of the amount of publication is the easiest (perhaps the only) means that is available from a statistical standpoint. There is a great deal of information contained in Graph II and Tables III–V concerning this fundamental matter of the amount of publication, and, in spite of the reservations just made, the data have real significance.

Table III gives the numbers and percentages of persons who have published no research articles, one, two, three, etc. research articles; the first division of the table gives figures for all Ph.D.'s granted in 1862–1933 and the second (we note an improvement) for those during the central period 1895–1924, which was selected as being far enough in the past so that men have had an opportunity to get something into print, and not extending back far enough so that lack of publication facility and of stimulus enters into the calculation.

A perusal of the figures in the various tables indicates that not more than one-third of the persons taking doctor's degrees have made as substantial contributions to research as would be evidenced by the publication of three or more research articles; and that not more than one-fifth have really been consistently productive in their contributions. About 60 (or 5%) of the doctors are responsible for half of the published pages of research.

TABLE IV

STATISTICS REGARDING THE NUMBER OF PAPERS PUBLISHED BY AMERICANS WHO TOOK DEGREES 1862–1933

Group	No papers	1 paper	2 papers	3–5 papers	6–10 papers	11–20 papers	21–30 papers	30+ papers
	%	%	%	%	%	%	%	%
American degree								
–1894	38	18	8	18	5	8	2	3
1895–1899	54	7	19	5	4	4	2	5
1900–1904	30	22	3	7	13	10	7	6
1905–1909	29	20	7	19	5	12	3	4
1910–1914	37	17	13	12	7	8	3	3
1915–1919	40	19	8	11	10	7	2	1
1920–1924	46	19	9	11	3	7	2	2
1925–1929	45	18	6	14	10	4	1	1
1930–1933	56	23	9	9	1	1	0	1
Foreign degree								
–1894	16	5	0	5	32	11	21	11
1895–1899	8	25	8	0	0	25	17	17
1900–1904	8	15	0	23	0	31	15	8
1905–1909	0	30	20	30	10	10	0	0
1910–1914	10	0	0	10	10	50	10	10

A more careful analysis of this situation in Table IV shows salient facts as follows: the percentage of persons doing no publishing has in recent years slightly increased; the percentage of persons publishing a large number of

papers has remained fairly constant in the group of those taking American degrees as well as of those taking European; the proportion of prolific authors is greater among those taking degrees abroad. Contrary to the general opinion, America seems in recent years to be adding to the quantity of personnel, but not improving the average quality as judged by the number of papers published.

It would be exceedingly interesting to know what proportion of the men and women with mathematical ability of high order are now being drawn into the graduate schools; in other words, how efficiently the nation is using this human material. Are there more persons of mathematical talent in the nation than can well be utilized? But such a study, which would have to begin with high school students, is entirely beyond the power of any single organization such as the Mathematical Association of America or the American Mathematical Society.

TABLE V

AMOUNT OF PUBLICATION BY GROUPS OF FIVE YEARS AFTER RECEIVING DEGREE

Period	First 5 years	Second 5 years	Third 5 years	Fourth 5 years	Fifth 5 years	Sixth 5 years	Seventh 5 years	Total	
American degree	pages	pages	pages	pages	pages	pages	pages	papers	pages
–1894	14.91	8.92	7.38	8.75	4.51	7.03	6.25	6.32	57.75
1895–1899	19.43	16.24	17.50	9.72	15.74	7.39	11.70	7.02	97.72
1900–1904	33.40	26.67	30.80	17.58	9.31	13.33		7.15	131.07
1905–1909	34.07	30.30	12.78	13.80	8.94			5.76	89.89
1910–1914	30.18	19.70	18.31	17.29				6.63	85.47
1915–1919	18.61	10.75	13.66					3.57	43.01
1920–1924	27.40	19.18						3.65	46.58
1925–1929	34.02							2.97	34.02
Foreign degree									
–1894	13.95	44.32	43.74	23.63	30.68	18.26	25.37	14.45	199.95
1895–1899	50.00	47.17	60.00	25.25	19.00	15.00	8.37	16.75	224.79
1900–1904	45.54	23.15	21.85	15.77	15.92	13.69		11.70	135.92
1905–1909	24.10	4.50	4.70	8.50	3.90			3.80	45.00
1910–1914	39.20	33.40	45.10	90.60				17.20	208.30

In Table V the amount of publication is analyzed by periods of five years after receiving the degree, and it should be noted that figures are given for a five-year period and that no average per year is tabulated. It should further be observed that in this table the figures pertain to pages instead of to papers published. As an example of how they are to be interpreted, we note that for all the doctors receiving American degrees for the five-year period 1895–99, the average number of pages which they published during the second five years

after receiving degrees was 16.24, which implies that the index, or average number of pages published annually, was 3.25.

For statistical purposes those papers which were printed before the degree was acquired are included in the first period. The average length of paper over the whole period is approximately 13.48 pages (4,916 papers and 66,268 pages*) and does not seem to vary significantly either with the age of the writer or with the date at which he took his degree. The first period of five years seems to be the most prolific one; doubtless this is due to the fact that men printing only one paper contribute during this period only. It is noteworthy that, even after more than thirty years subsequent to the conferring of the degree, there is on the part of many research workers not much slackening in the rate of publication.

The fifth edition of *American Men of Science* (1933) contains brief biographies of 1,242 living mathematicians who are listed because they "have contributed to the advancement of science." In the earlier editions there were listed an additional 168 names of people since deceased. Of this total of 1,410 mathematicians, 934 hold the doctor's degree; 104 of these degrees were awarded abroad; and of the recipients, 59 were born abroad Since *American Men of Science* contains primarily the names of persons now in residence in the United States and only incidentally a few of those working in Canada, the information has not so wide a basis as that in the study presented here, which includes Canada also.

On the basis of extensive ballots, Professor Cattell, in the various editions of *American Men of Science*, selected a total of 182 mathematicians (136 were listed as alive in the fifth edition) as representing the leaders in research, and stars were attached to their names. Of this group all but 19 have obtained the doctor's degree; 40 were born in Europe; and 56 received their doctor's degrees abroad. Our present study indicates that these 182 mathematicians have an average index of 13.03 as compared to 4.73 for the whole list of doctors, and that approximately 45,580 pages, or 69% of the whole mathematical publication of America, is due to them.

Comparison of Productivity as Regards Universities of Origin

Since it was the more enterprising and able of our students who went abroad during the twenty-year period beginning with 1895, it is to be expected that the productivity of the group taking their degrees in European universities would be greater than that of the corresponding group awarded American degrees. Graph II, which gives the average number of pages printed yearly during a period of not more than thirty-five years subsequent to attaining the degree, indicates that, for a period of twenty years before the world war (except for a lapse during 1905–09), the European universities attracted a considerable number of our ablest men as candidates for degrees. So far as the home product is concerned, we note that a peak was reached with the group of men taking degrees in 1920–24.

* This total is for Americans holding doctor's degrees and does not include articles presented to the Society by foreigners or non-doctors.

A department in a university waxes and wanes. At one period an outstanding professor attracts an able body of students and inspires them to continue their research; at another the quality of the staff and students is mediocre. If we consider a group of ten institutions, each conferring a large number of degrees, we note from Table VI that there is very considerable variation in the matter of productivity. This is true both as between the universities and in a given university as regards the four decades covered by the table. The figures in parentheses represent the number of degrees conferred in the period named in the heading of the column, and the other figures represent the average number of pages per person per year subsequently published. It is a matter of consider-

TABLE VI

COMPARATIVE STUDY OF AMOUNT OF PUBLICATION PER GRADUATE IN TEN INSTITUTIONS

Institution	1890–99	1900–09	1910–19	1920–29	Whole period
1	(4) 26.63	(35) 6.69	(56) 3.31	(90) 4.28	(185) 4.92
2	(16) 0.95	(18) 4.62	(21) 1.91	(23) 0.96	(78) 2.06
3	(5) 1.50	(12) 4.59	(26) 6.33	(27) 13.96	(70) 8.63
4	(13) 0.56	(23) 2.55	(15) 2.15	(10) 4.86	(61) 2.41
5	(2) 0.00	(15) 4.87	(16) 1.01	(24) 4.06	(57) 3.28
6		(1) 0.56	(14) 1.87	(35) 1.21	(50) 1.38
7	(4) 6.36	(11) 0.69	(21) 8.21	(11) 5.45	(47) 5.65
8	(9) 0.02	(9) 2.34	(12) 1.29	(11) 17.34	(41) 5.55
9	(1) 0.44	(3) 0.92	(11) 8.71	(13) 12.58	(28) 9.38
10			(9) 1.15	(11) 9.05	(20) 5.50
Average of the ten	3.01	4.22	3.77	5.83	4.62
General average of all Ph.D.'s		4.34	3.60	5.68	4.73

able significance that the persons taking degrees at these large institutions (and constituting more than three-quarters of the total during that period) do slightly less publication on the average than those taking their degrees at the remaining group of institutions. It is noteworthy also that the average of the indices for some of the institutions not included in this list of ten is higher than for any in the list.

Persons Without Degrees

Many persons who have not attained to doctor's degrees have published papers read before the Society. The number of such persons is about 125, and a few of them are ranked among our leading mathematicians.

National Research Fellows

It is to be expected that the publication rate for the National Research Fellows in mathematics will be greater than that for the whole group of per-

sons taking degrees, not only because they have been given special opportunities, but also because they are a picked body of men. Let us select as a basis for discussion the 49 Fellows starting on their fellowships during the first five-year period, 1925–29, in which the fellowships were awarded. The average number of pages published annually over a five-year period subsequent to their entering on the fellowship is 17.82, while for all 227 taking Ph.D.'s during the years 1925–29, it is 6.80. For the period of tenure (generally two years) and one year thereafter (to allow for lag in publication), the average for the Fellows is 25.89; this indicates what can be expected from young men if a proper selection for quality is imposed and opportunity for full-time study is made available.

For accurate comparison purposes, however, it is desirable to have more equitable bases than those just used. Let us eliminate from the 49 Fellows starting on their incumbencies in 1925–29 the 4 persons who took their degrees earlier than 1923 and who had already thus had opportunity to establish themselves in research careers. Let us allow a lapse of a year and a half after giving up the fellowship; the doctor is then professionally employed and has probably already published the main results of the research done while an incumbent. For the other group let us select those Ph.D.'s of the same period who were not awarded fellowships and let us allow a lapse of a year and a half in their cases also, so that the doctoral thesis will probably not be included in the publications counted; let us compare these groups during the subsequent years and down to the time of the last information available (including 1934). The figure for the Fellows is 10.06 and for the remaining Ph.D.'s is 2.87, which establishes a significant differential. However, using the same groups and the same periods, let us compare the bottom one-third of the Fellows with the top one-third of the non-fellows. The results are 0.93 for the 15 persons of the one group as against 8.49 for the 60 persons of the other, indicating that the selection of the Fellows carried with it a considerable factor of error. It should, however, be pointed out that not all the more talented students apply for fellowships.

Let us next make a comparison of the 30 most productive of the 45 Fellows considered above and the 30 most productive Ph.D.'s among the 180 non-fellows of the period 1925–29 by subjecting their records to the conditions of the preceding paragraph. The indices are 14.63 and 14.77; but it must be pointed out that these selections are for the upper two-thirds and upper one-sixth respectively.

Another fundamental comparison that can be made is that between the Fellows of the 1925–29 period and the corresponding persons of the 1920–24 period before the fellowships were available. Selecting the 30 from each group who have published the most, and using comparable periods beginning one and one-half years after fellowship and degree respectively and terminating with 1934 and 1929 respectively, the figure for the Fellows is 14.63 and for the others is 10.16. Since in this comparison the case for the Fellows is discounted by the fact that not all the best men are elected to fellowships, the figures indicate a decided stimulus from the fellowships.

Conclusion

The rapidity with which a mathematical school of high distinction has been built up in America is one of the most striking phenomena in the history of science. It should be borne in mind that the American Mathematical Society, which has been an important factor in this development, was not founded until 1888, and that only in isolated cases was research carried on before that date. Building on the splendid foundations already laid, great forward movements are possible if the spirit of cooperation now animating mathematicians is fostered. The challenge of the future inspires the mathematical fraternity to high endeavor.

Many of the statistics exhibited in this paper will be of interest to those who follow the development of teaching and research in the field of mathematics. Those concerned with the strategy of promoting mathematical thought and achievement in America can find in the assembled material several sign-posts for their further guidance. Other studies could be made from the data which have been collected and which are available to anyone; the investigations appear to be worth pursuing further.