

# Theodore von Kármán and Applied Mathematics in America<sup>1</sup>

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Applied mathematics is generally regarded as having become a distinct discipline in the United States during World War II. Brown University, under Roland G. D. Richardson, formally instituted a program in applied mathematics, the nation's first, in 1941. New York University, under Richard Courant, later established its own program (1). By that time, Theodore von Kármán (1881–1963), Hungarian-born engineer and applied scientist and the first director of the Daniel Guggenheim Graduate School of Aeronautics at the California Institute of Technology, had already spent more than 10 years struggling to make applied mathematics respectable in his adopted country. To him, the measures taken during the war represented the first concerted, nationwide effort to resolve a long-standing scientific gap in the United States.

Von Kármán figured prominently in the rise of Caltech's school of aeronautics in the 1930s and his experience in America in the 1930s helped define the issues that would lead to the organized development of applied mathematics in the next decade. Frequently pressed for his opinions on how to mobilize mathematicians for the war, von Kármán contributed the lead article "Tooling up mathematics for engineering," to the first issue of the *Quarterly of Applied Mathematics*, published in 1943 (2) under the auspices of Brown's program. Using the form of a dialogue, he eloquently stated the case for the applied mathematician in the service of science. He did not, however, wholeheartedly approve of the proposals for new applied mathematics institutes drafted just before Pearl Harbor, especially the "exaggerated" appeal to an "emergency" created by the war. In his review of one such proposal, he noted that the problem of applied mathematics could not be solved "through the ordinary process of supply and demand" (3,4). Indeed, an entirely different set of imperatives guided von Kármán in the 1930s.

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## MATHEMATICIANS AND ENGINEERS

Shortly after he had completed his first tour of the United States in 1926, which included a visit to Caltech, von Kármán wrote to Courant, then the head of Göttingen's mathematics institute, that "what strikes me most in regard to mathematics...[in America] is the complete lack of 'applied' mathematicians . . ." (5,6). In one sense, von Kármán certainly erred; since the late 19th century, electrical and radio engineering had evolved into highly advanced branches of applied science in the United States, involving the use of a great deal of sophisticated mathematics, thanks to the efforts of several dozen applied scientists, including Charles P. Steinmetz, Michael I. Pupin, and Frank Jewett (7).

At the same time, there was some truth in von Kármán's perception: pure mathematics had developed by leaps and bounds in the United States during the early part of the 20th century. The contact that aspiring American mathematicians had with certain European mathematical schools, especially the German abstract school, which underwent rapid development during the latter half of the 19th century, provided the initial stimulus. As a result, American mathematicians were able, within a short time, to build and sustain research groups in American academic settings in the areas of analysis, number theory, and especially a new branch of mathematics, topology (8).

Contemporary historians of mathematics also emphasized the origins of pure mathematics. R. C. Archibald, for example, reported in 1925 that pure mathematical research in American universities began with Benjamin Peirce (1809–1880) (9). In fact, less than half his output is considered pure mathematics today; the applied mathematics Peirce did all but escaped Archibald's attention.

Except for electrical engineering, von Kármán's reading of the state of American applied science was by and large accurate. The science of the strength of materials, for example, von Kármán's first field of study, remained almost exclusively experimental in the United States. American engineers, generally pragmatic, distrusted the increasingly sophisticated theoretical and mathematical formulations overseas. Mathematically unsolvable problems had no place in turn-of-the-century American engineering practice (10). This may help explain why Russian-born Stephen Timoshenko, sometimes called the "father of engineering mechanics" in the United States, was virtually unknown to the American scientific community when he arrived in 1922 at the age of 44. In time, his reputation in the Old World reached the new one (11). According to Timoshenko, who went to work for Westinghouse in 1923, "all the jobs requiring any theoretical knowledge whatsoever were filled mainly by engineers educated in Europe" (11, p. 248). While designing machinery at Westinghouse, he also taught elasticity theory to the other engineers, probably

the first such course in the country (11, p. 252; 12). After moving to the University of Michigan in 1927, Timoshenko initiated a program in engineering mechanics similar to one von Kármán would bring to Caltech. Von Kármán, in fact, insisted that Timoshenko had made “the first attempt to gather the applied mathematicians and to institute some activity in applied mathematics” in the United States (5). Both men complained about the attitude of American engineering students. In Timoshenko’s opinion, they only wanted “the final result — a formula which . . . [they] can apply mechanically, without thought, to solve practical problems.” He traced this attitude back to inadequate mathematics instruction in American high schools (11, p. 26). Indeed, during the 1930s, mathematics came under constant attack, especially at the high school level (13). Engineering educators well into the 1920s had seriously debated whether engineering students even ought to study calculus. Some thought that such courses were mere “cultural embellishments to the curriculum” (14).

Von Kármán did not find mathematicians at Caltech particularly helpful either. Number theorist Eric Temple Bell, for instance, was not interested in training engineers, and the mathematics learned in Bell’s hands, according to von Kármán, was simply too abstract (15, p. 149). Von Kármán felt strongly that applied mathematics should be taught in graduate engineering schools, but this seldom happened. The mathematicians told the engineers to teach the course, but the engineers concentrated only on practical subjects (16).

Von Kármán had his feet planted in both worlds. He had done work on the buckling of columns, on the stability of vortex patterns that form behind stationary bodies in flowing fluids, and, with Max Born, on the lattice dynamics and vibrational frequencies of crystals, advancing work done earlier by Albert Einstein and Peter Debye on the heat capacity of solids, among other things. He brought a mathematically sophisticated point of view to all of these problems. Yet when he arrived in the United States, he found American engineers largely untutored in certain branches of mathematics, and quite unprepared for his unorthodox approach to the engineering sciences.

### AERONAUTICAL TRADITIONS AND INNOVATIONS

Although aeronautics at Caltech is often said to have begun when von Kármán arrived in 1930, its roots go back to the formation of a committee on aeronautics at Throop College (later renamed Caltech) in 1917, as the United States prepared to join the war against Germany. Throop’s science-minded trustee George Ellery Hale promoted aeronautics research as a way for the school to gain national stature. The college hired Harry Bateman, an English mathematical physicist, and Albert A. Merrill, an American inventor. Bateman was the theoretician, Merrill the tinkerer. After designing a

small wind tunnel for testing models, Merrill began work on a plane design featuring a movable wing.

By the mid-1920s, Bateman had acquired several graduate students, including Clark Millikan, the son of Caltech physicist Robert A. Millikan. Following von Kármán's first visit to the campus in 1926, Clark Millikan kept him informed by mail of what was going on in aeronautics until von Kármán returned to Pasadena permanently in 1930. The letters told of the construction of the aeronautics laboratory, breaking in and experimenting with the 10-foot Göttingen-style tunnel von Kármán had urged the school to construct, Merrill's new airplane, and Bateman's recent work on airfoil theory (17).

Merrill, a self-taught inventor well versed in the practical side of aeronautics, had the field to himself at Caltech until Robert Millikan, the school's head, engaged Arthur E. Raymond, a member of the technical staff of Douglas Aircraft Company and an expert in designing planes, to teach a class in aircraft design. Merrill left Caltech before von Kármán became director of the Guggenheim Aeronautical Laboratory. The 1928 crash of Merrill's biplane, "the dill pickle" as his students called it, may have hastened his departure.

Bateman specialized in finding particular solutions to complicated equations used by physicists and applied mathematicians. In contrast to Merrill, Bateman was shy and unassuming. Indeed his Caltech colleague E. T. Bell, fearful that Bateman might shortchange his chances of election to the National Academy of Sciences by listing too little on his curriculum vitae, counseled, "Spread yourself; it pays, in our glorious country, to kick over the bushel and let your light to shine before men that they may see your good works . . ." (18).

In Merrill and Bateman's time, the aviation field still belonged to amateurs, and Merrill was high in their ranks. In contrast, von Kármán's students and co-workers attacked a host of theoretical problems related to airplane design and flying that industry used to good advantage (19). The presence of Raymond on the campus indicates that the southern California-based aircraft companies and Caltech had discovered each other before von Kármán took up permanent residence in the United States. There is little doubt that the companies profited even more from the creation of a first-class school of aeronautics in Pasadena. Indeed, records from the Guggenheim Aeronautical Laboratory reveal that of the 30 most prominent graduates in the 1930s nearly half — those who were theoretically oriented — joined universities and the others worked in industry, especially the local aircraft companies. In general, students who did their work in aerodynamics went into the aircraft industry, and those who specialized in fluid mechanics (studying problems such as turbulence and the boundary layer) became academics (20). The 10-foot wind tunnel at the Guggenheim Laboratory, designed to von Kármán's specifications, was used to test practically all the aircraft built by the companies on the West Coast during the 1930s, including the Douglas Company's

DC-3 series, the most successful commercial aircraft of the time. The aircraft companies also recruited Caltech's outstanding students. W. Bailey Oswald, who received his doctorate in 1932, was hired by Raymond as Douglas' chief aerodynamicist when the company began working on the DC-1. The relation between the Guggenheim Laboratory and local aircraft companies in the 1930s foreshadowed the rapid development during World War II of an academic-industrial complex.

Von Kármán's aeronautics school of the 1930s directly benefited from the European applied mathematics and mechanics movement of the 1920s and von Kármán's participation in it. That movement found expression in new organizations, journals, and academic departments. As head of Aachen's Aerodynamics Institute in Germany as well as professor of aerodynamics and mechanics, von Kármán took the initiative in organizing the 1922 conference on hydro- and aerodynamics in Innsbruck, Austria. The 4-day meeting, boycotted by French and British scientists, attracted 33 applied mathematicians and physicists from seven European countries. This informal post-World War I conference meeting, for which von Kármán personally divided the organizing costs with Italian mathematician Tullio Levi-Civita, succeeded in bringing together a number of people with similar scientific interests for the first time. Von Kármán saw that aerodynamicists like himself did not get the attention they deserved because there were not enough of them to stand out at ordinary scientific meetings. "And even among the group they are very split," he pointed out, "because the mathematicians attend mathematics meetings, the physicists attend physics meetings, and the technical people go only to technical meetings" (21). Von Kármán belonged to a group of scientists who decided to do something about the problem. Innsbruck was his solution.

A contemporary, Richard von Mises, founded in 1921 and edited a new journal for applied mathematics and mechanics (*Zeitschrift für angewandte Mathematik und Mechanik*). The head of Berlin's Institute for Applied Mathematics, itself a post-World War I development, von Mises had a flair for organizing like-minded scientists. According to von Kármán, it was von Mises who first mobilized physicists, mathematicians, and scientifically minded engineers working on applied problems to publish their results in the same place (22).

Von Kármán also disregarded the traditionally defined boundaries for aeronautics and aerodynamics in the United States. The range of problems he tackled encompassed more than either science usually did. Robert Millikan's criticism of the state of American engineering paved von Kármán's way at Caltech. Millikan had singled out "the 'ad hoc approach' to the practical problems to be solved" as the weak link in the nation's engineering schools (23). "If a man does not learn his physics, chemistry, and mathematics in college, he never learns it," he told a Caltech audience in 1920, adding, "the attempt to learn the details of an industry in college is futile. The industry

itself not only can, but it must, teach these" (24). Although von Kármán did not succeed in converting all Caltech's engineers to his point of view, the institute's philosophy nevertheless provided him with the necessary freedom to pursue his own course.

In 1932, the Metropolitan Water District of Southern California asked von Kármán for help in designing pumps for its Colorado River aqueduct project. In petitioning the school to establish a hydraulics laboratory, von Kármán likened the state of hydraulics to that of aeronautics when engineers first began to turn away from purely empirical computations and started to embrace the methods of the applied mathematicians (25). In the "pump lab," as it came to be called, Caltech's engineers, von Kármán among them, studied a variety of water flow problems. Among other things, they designed and built a water "wind tunnel" to test the efficiency of various pumps. The work done in the hydraulics laboratory, von Kármán once said, "showed a generation of engineers how pure scientific ideas in hydrodynamics, aerodynamics, and fluid mechanics can be used to solve problems of practical design in related fields that at first seem remote" (15, pp. 205–206).

Von Kármán also had a hand in the Grand Coulee Dam project. When cracks appeared after the dam opened, von Kármán realized almost immediately that the forces on the dam exceeded the buckling limits for which it was designed. What the civil engineers had done, in effect, was to use standard design factors obtained from a handbook and then extrapolate to get the figures for building a dam the size of the Grand Coulee. Although they had taken into account static forces due to water pressure acting on the dam, they had failed to consider the special buckling conditions that would arise in such a large dam (15, pp. 207–208). Von Kármán advised the dam engineers to put in stiffeners, drawing on his experience with stiffeners in making sheet metal usable in aircraft design to solve a civil engineering problem.

In another nonaeronautical assignment, von Kármán and his co-workers solved the mystery of "Gallopig Gertie," the collapse of the Tacoma Narrows Bridge in 1940. In characteristic fashion, von Kármán transformed a statics problem in civil engineering into a dynamic instability problem. The solution rested on an appreciation of a complex hydrodynamic phenomenon known as vortex shedding first explained by von Kármán in 1911 (26). In recalling the episode many years later, von Kármán noted that "the bridge engineers couldn't see how a science applied to a small unstable thing like an airplane wing could also be applied to a huge, solid, nonflying structure like a bridge" (15, p. 214). In all these instances, it was von Kármán, the

applied mathematician, who was able to see the solution by cutting across the boundaries of the traditional engineering fields.

### APPLIED MATHEMATICS VERSUS MATHEMATICAL PHYSICS

Often, neither the mathematicians nor the engineers grasped the role of mathematics in applied science. Von Kármán continually pointed out the difficulties mathematicians and physicists had in dealing with nonlinear problems, where intuition alone would not suffice (27). In some instances, the mathematics to deal with such problems had not yet been invented, as was the case with the solitary wave problem (28), the forerunner of solitons, the mathematics of which physicists struggle with today. Mathematicians preferred to deal in generalities, seldom taking, in von Kármán's words, "the pains to find and discuss the actual solutions," except in the simplest cases (29).

Von Kármán continually stressed the difference between mathematical physics and applied mathematics. Once, to make his point, he compared working in applied mathematics to shopping in "a warehouse of mathematical knowledge." The scientist could live in the warehouse and find uses for the equations on the shelf, or he could visit the place from time to time with a shopping list. Von Kármán saw himself as a shopper, not as the caretaker of the mathematics building (30). To a rigorous mathematical physicist like John L. Synge, however, von Kármán's style left something to be desired. Writing to H. P. Robertson, a colleague at Princeton, Synge said, "Kármán has a wonderful intuition, but to a mathematician his exposition is appalling; I think you know that already" (31).

Von Kármán and Courant, who emigrated to the United States in 1934, did not see eye to eye on the development of applied mathematics. Courant was fundamentally interested in mathematical physics (32, p. 226). He used mathematics to make the underpinnings of physics more rigorous. In lectures on this subject, he discussed mathematical problems which had their roots in classical physics. Unlike von Kármán, who used mathematics to solve physics problems, Courant stressed general theories. The algebra and analysis in the 1924 textbook on mathematical physics (33), coauthored by Courant, later provided physicists with tools for further developing quantum mechanics, despite the book's classical physics origins (32, p. 98 and pp. 113–114). Because physicists incorporated some of this mathematics into their own work, Courant saw his role in this work as that of an applied mathematician. But von Kármán would not have defined Courant in that way; Courant, in his view, was really preoccupied with the kinds of questions mathematicians ask, not those of applied scientists. More often than not, von Kármán and his colleagues had to devise their own mathematics of approximate solutions in working out specific technical problems (29).

Caltech's Bateman illustrates another aspect of mathematical physics. Bateman used his mastery of partial differential equations to push Maxwell's equations of electrodynamics to their limits. During the early 1920s, he applied his considerable mathematical skills to devise ingenious theories of radiation to account for the Compton effect, in an effort to save classical physics (34). Paul Ehrenfest, a visitor at Caltech in 1923, marveled at Bateman's uncanny ability but was not persuaded that the mathematician grasped the physics that underlay his calculations. In describing how they wrote a paper together, Ehrenfest remarked: "By my completely desperate questioning, I chased him around for so long in the primeval forest of his calculations that the thing grew clearer and clearer. The connections among his curious isolated results stood out ever more sharply (for him, too!!)" (35). Bateman, to use von Kármán's metaphor, lived in the mathematical warehouse. He had little in common with applied mathematicians, whom he described on one occasion as mathematicians "without mathematical conscience" (36).

Von Kármán's approach to applied mathematics reflected those of people like von Mises and Hugh Dryden. He especially admired Dryden, who like Raymond, belonged to that small band of early American aviation enthusiasts with solid backgrounds in physics and mathematics. At the age of 20, Dryden earned his Ph.D. from Johns Hopkins in 1919, with an experimental thesis on airflow. As chief of the Aerodynamics Section in the National Bureau of Standards, Dryden continued to work on airflow problems, including turbulence and the boundary layer. In 1941 he succeeded J. C. Hunsaker as the editor of the *Journal of the Aeronautical Sciences* (37).

## PUBLISHING

There was no American journal in the 1930s comparable to Mises' *Zeitschrift* to publish applied mathematics papers. Moreover, the banding together of engineers by specialty hindered the founding of interdisciplinary journals. Von Kármán described the problem in a letter to Harvard professor Den Hartog: "American engineers are organized in separate societies. Mechanical, civil, electrical, aeronautical, and automotive engineers have their own organizations, and very little contact exists between them" (38). Each engineering society had a separate journal. Applied mathematics issues, essentially interdisciplinary papers, had to find space in existing journals. A key problem was finding sympathetic editors to deal with manuscripts that straddled more than one discipline. In von Kármán's opinion, only the *Journal of the Aeronautical Sciences* had "the proper attitude for theory" and not a "panicky fear of mathematics" (39). This engineering journal only came into existence in 1933 as the publication arm of the newly founded Institute of Aeronautical Sciences.



Throughout the 1930s few publication outlets were available for topics in applied mathematics. “Many papers are undoubtedly misplaced,” von Kármán wrote Brown University Dean Richardson in 1942, because of the way the societies and their journals were organized. Here was the proof, if any was needed, he told Richardson, of a niche “for such a [new] journal” (40).

During World War II applied mathematics turned into a subject for national debate over what the scope, objectives, and theory-to-practice ratio of the program at Brown University should be, as well as what to call the new journal to be published through the program. Some thought the journal should be the sequel to the von Mises *Zeitschrift*. Others thought the word “mechanics” ought to be dropped from the title. The final choice was the *Quarterly of Applied Mathematics*. Still one critic felt that the words “applied mathematics” had “no generally accepted meaning” (41) even in 1942. In some sense the difficulties that von Kármán faced in the 1930s had come to a head.

## CONCLUSION

Von Kármán encountered many obstacles during the 1930s beyond a lack of appropriate journals. Some of his novel solutions to structural and civil engineering problems, for instance, were looked at askance by civil engineers, despite Timoshenko’s pioneering work. Many older engineers were initially skeptical of von Kármán’s proposal to build a water wind tunnel in connection with the Colorado River aqueduct project. When a number of experts, including several Caltech civil engineers, could not solve the mystery of the cracks in the Grand Coulee Dam, and von Kármán was called in as a last resort, some protested, “but he has no civil engineering experience” (15, p. 207). When von Kármán recommended testing a model of the new Tacoma Narrows Bridge in a wind tunnel, even the eminent civil engineer O. H. Ammann said: “You don’t mean to say that we shall build a bridge and put it in a wind tunnel?” Von Kármán later noted that the builder of New York’s George Washington Bridge “knew better, but long tradition was dictating his remarks” (15, p. 214). The structural engineers assigned to investigate the collapse of the bridge simply found it hard to get beyond their deeply held beliefs in static forces.

The obstacles placed in the path of applied mathematics were probably no greater than those placed in the path of other interdisciplinary endeavors. Subjects ranging from physical chemistry and astrophysics, at the turn of the century, to biophysics and bioengineering in more recent times, have successfully bridged several disciplines and become independent enterprises. In all cases, skepticism from co-workers in the traditional fields appears to

be part of the natural-selection process. Under what circumstances some become independent disciplines and others do not, is poorly understood.

#### REFERENCES AND NOTES

1. World War II as a stimulus for applied mathematics is discussed by M. Rees [*Am. Math. Mon.* **87**, 607 (1980)]. N. Reingold [*Ann. Sci.* **38**, 313 (1981)], and J. B. Rosser [*Am. Math. Soc. Not.* **29**, 509 (1982)].
2. T. von Kármán, *Q. Appl. Math.* **1**, 2 (1943).
3. Theodore von Kármán Papers, Milikan Library, California Institute of Technology, Pasadena.
4. T. von Kármán, letter of 13 September 1941 to M. Moore (3, box 70, 11).
5. T. von Kármán, letter of 14 February 1927 to R. Courant (3, box 6, 14).
6. P. A. Hanle [*Bringing Aerodynamics to America* (MIT Press, Cambridge, Mass., 1982), chapters 3–6] describes von Kármán's experiences in Germany.
7. See C. Susskind [*Twenty-five Engineers and Inventors* (San Francisco Press, San Francisco, 1976)] for biographical sketches of these and other applied scientists.
8. G. Birkhoff, in *The Bicentennial Tribute to American Mathematics, 1776–1976*. D. Tarwater, Ed. (Mathematical Association of America, Washington, D.C., 1977), pp. 25–78.
9. See P. S. Jones, in *The Mathematical Association of America: Its First Fifty Years*, K. O. May, Ed. (Mathematical Association of America, Washington, D. C., 1972), p. 5.
10. E. Layton, *Technol. Culture* **12**, 573 (1971).
11. S.P. Timoshenko, *As I Remember*, R. Addis. Transl. (Van Nostrand, Princeton, 1968), p. v.
12. C. R. Soderberg, *Biog. Mem. Natl. Acad. Sci.* **53**, 335 (1982).
13. C. B. Boyer, in *The Mathematical Association of America: Its First Fifty Years*, K. O. May, Ed. (Mathematical Association of America, Washington, D. C., 1972), p. 32.
14. E. T. Layton, *The Revolt of the Engineers: Social Responsibility and the American Engineering Profession* (Case Western Reserve Univ. Press, Cleveland, 1971), p. 4.
15. T. von Kármán with L. Edson, *The Wind and Beyond* (Little, Brown, Boston, 1967).
16. T. von Kármán, letter of 16 March 1937 to R. Courant (3, box 6.15).
17. The Clark B. Millikan Papers (Institute Archives, California Institute of Technology, Pasadena) are a rich source of information about the period before von Kármán's arrival.
18. E. T. Bell, letter of 4 July 1927 to H. Bateman, (Harry Bateman Papers, Institute Archives, California Institute of Technology, Pasadena), box 1.1.
19. W. R. Sears and M. R. Sears, *Annu. Rev. Fluid Mech.* **11**, 7 (1979).
20. See the *Bull. Calif. Inst. Technol.* **49** (No. 2) (1940) and Caltech aeronautics department statistics on employment of students after graduation.
21. T. von Kármán, letter of 12 April 1922 to T. Levi-Civita (3, box 18.8).

22. —, letter of 8 September 1942 to R. G. D. Richardson (3, box 80.38); letter of 16 February 1945 to H. M. Westgaard (3, box 20.37).
23. —, and F. L. Wattendorf, in *Miszellanneen der angewandten Mechanik, Festschrift Walter Tollmien* (Akademie-Verlag, Berlin, 1962), p. 103.
24. R. A. Millikan assembly address of 6 January 1920 (Robert A. Millikan Papers, Institute Archives, California Institute of Technology, Pasadena), box 20.9, pp. 4–5.
25. T. von Kármán, a proposal to establish a hydraulics laboratory at the California Institute of Technology, 11 January 1932 (3, box 72.10).
26. A. F. Gunns, *Pac. Northwest Q.* **72**, 168 (1981).
27. T. von Kármán, *Bull. Am. Math. Soc.* **46**, 617 (1940).
28. —, *ibid.*, p. 60.
29. —, *Q. Appl. Math.* **1**, 2 (1943).
30. —, letter of 8 September 1942 to R. G. D. Richardson (3, box 80.38).
31. J. L. Synge, letter of 7 April 1941 to H. P. Robertson (Howard P. Robertson Papers, Institute Archives, California Institute of Technology, Pasadena), box 4.47.
32. C. Reid, *Courant in Göttingen and New York: The Story of an Improbable Mathematician* (Springer-Verlag, New York, 1976).
33. D. Hilbert and R. Courant, *Methoden der Mathematischen Physik* (Springer, Berlin, 1924).
34. R. H. Stuewer, *Proc. Int. Congr. Hist. Sci.* **2** 320 (1975); *The Compton Effect: Turning Point in Physics* (Science History Publications, New York, 1975), pp. 312–314.
35. P. Ehrenfest, letter of 16 February 1924 to T. Ehrenfest, quoted in M. J. Klein, “America observed: Paul Ehrenfest’s visit in 1923–24,” paper presented at the Conference on the Recasting of Science between the Two World Wars, Florence and Rome, 1980.
36. M. A. Biot, *J. Aeronaut. Sci.* **23**, 406 (1956).
37. T. von Kármán, letter of 24 February 1940 to J. P. Den Hartog (3, box 7.2); letter of 8 September 1942 to R. G. D. Richardson (3, box 80.38).
38. —, letter of 24 February 1940 to J. P. Den Hartog (3, box 7.2).
39. —, letter of 14 February 1940 to J. P. Den Hartog (3, box 7.2).
40. —, letter of 8 September 1942 to R. G. D. Richardson (3, box 80.38).
41. W. Prager, letter of 9 December 1942 to H. Dryden, in (3, box 80.38).
42. Supported in part by the Haynes Foundation.